SOLUTIONS TO AIEEE 2010

PART A – PHYSICS

Direction : Questions number 1 -3 are based on the following paragraphs.

An initially parallel cylindrical beam travels in a medium of refractive index $\mu(I) = \mu_0 + \mu_2 I$, where μ_0 and μ_2 are positive constants and I is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius.

1. The initial shape of the wavefront of the beam is
(1) planar
(2) convex
(3) concave
(4) convex near the axis and concave near the periphery

1. (1)
Wavefront is perpendicular to the direction of propagation of wave.

2. The speed of light in the medium is
(1) maximum on the axis of the beam
(2) minimum on the axis of the beam
(3) the same everywhere in the beam
(4) directly proportional to the intensity I

2. (2)
Near the axis refractive index will be maximum therefore velocity on the axis will be minimum

3. As the beam enters the medium, it will
(1) travel as a cylindrical beam
(2) diverge
(3) converge
(4) diverge near the axis and converge near the periphery

3. (1)
As the beam inters the medium it will travel as a cylindrical beam

Direction: Questions number 4 – 5 are based on the following paragraph.
A nucleus of mass $M + \Delta m$ is at rest and decays into two daughter nuclei of equal mass $\frac{M}{2}$ each. Speed of light is c.

4. The speed of daughter nuclei is

(1) $c\sqrt{\frac{\Delta m}{M+\Delta m}}$ (2) $c\frac{\Delta m}{M+\Delta m}$

(3) $c\sqrt{\frac{2\Delta m}{M}}$ (4) $c\sqrt{\frac{\Delta m}{M}}$

4. (3)

Energy released in decay process = $\Delta m\ c^2$

Here, K. E. of the daughter nuclei = $\Delta m\ c^2$

or, $\frac{1}{2}\frac{Mv^2}{2} + \frac{1}{2}\frac{Mv^2}{2} = \Delta m\ c^2$ [Both daughter nuclei will have same magnitude of velocity as per momentum conservation]

5. The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2. Then

(1) $E_1 = 2E_2$ (2) $E_2 = 2E_1$

(3) $E_1 > E_2$ (4) $E_2 > E_1$

5. (4)

Higher is the B.E. per nucleon, greater is the stability of nucleus and nuclear transformation proceeds to achieve stability.

$\therefore E_2 > E_1$

Direction: Questions number 6 - 7 contain Statement-1 and Statement -2. Of the four choices given after the statements, choose the one that best describes the two statements.

6. **Statement-1**: When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectron is K_{max}. When the ultraviolet light is replaced by X-rays, both V_0 and K_{max} increase.

Statement-2 : Photoelectrons are emitted with speeds ranging from zero to a maximum value because of the range of frequencies present in the incident light.

(1) Statement – 1 is true; Statement – 2 is false.

(2) Statement – 1 is true, Statement – 2 is true; Statement – 2 is the correct explanation for Statement – 1.

(3) Statement – 1 is true, Statement – 2 is true; Statement – 2 is not a correct explanation for Statement – 1.

(4) Statement – 1 is false; Statement – 2 is true.

6. (1)

7. **Statement-1** : Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

Statement-2 : Principle of conservation of momentum holds true for all kinds of collisions.

(1) Statement – 1 is true; Statement – 2 is false.

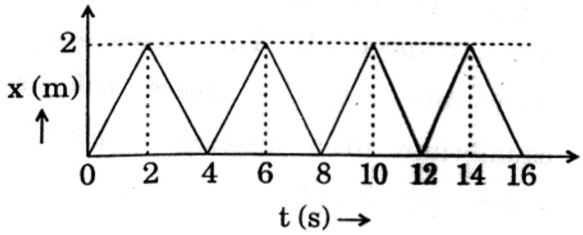
(2) Statement – 1 is true, Statement – 2 is true; Statement – 2 is the correct explanation for Statement –1.

(3) Statement – 1 is true, Statement – 2 is true; Statement – 2 is not a correct explanation for Statement – 1.

(4) Statement – 1 is false; Statement –2 is true.

7. (2)

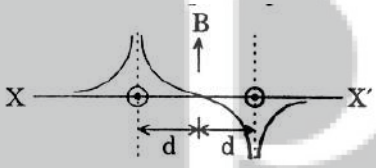
8. The figure shows the position –time (x - t) graph of one-dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is

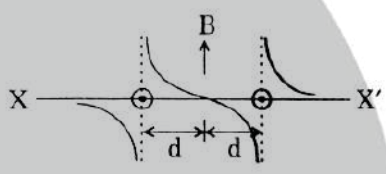


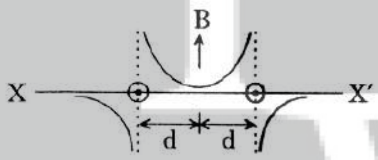
(1) 0.2 Ns
(2) 0.4 Ns
(3) 0.8 Ns
(4) 1.6 Ns

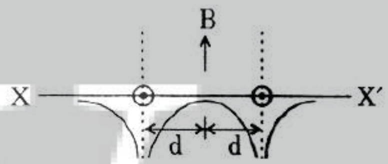
8. (3)
Impulse = change in momentum
I = 0.4 [1–(–1)]
I = 0.8 Ns

9. Two long parallel wires are at a distance 2d apart. They carry steady equal currents flowing out of the plane of the paper as shown. The variation of the magnetic field B along the line XX′ is given by

(1)


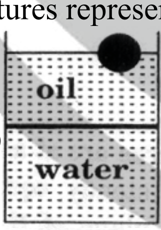
(2)


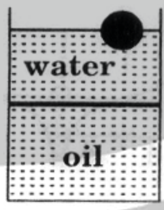
(3)


(4)


9. (2)
The magnetic field at the mid point is zero and will be along negative direction in left side of left wire but along positive direction in right side of right wire.

10. A ball is made of a material of density ρ where $\rho_{oil} < \rho < \rho_{water}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position?

(1)


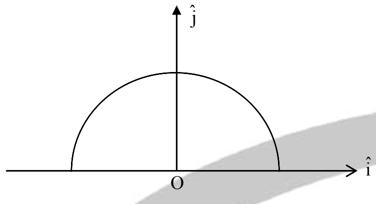
(2)


(3)


(4)


10. (3)
It cannot float only in oil for the given condition of densities clearly it will float at the junction having oil above water.

11. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. the net field $\vec{E}$ at the centre O is



(1) $\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$ (2) $\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$

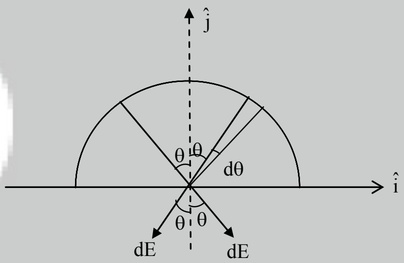
(3) $-\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ (4) $-\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$

11 (4)

$$\vec{E} = 2\int dE\cos\theta(-\hat{j})$$

$$\vec{E} = 2\int_0^{\pi/2} \frac{kq}{\pi r}\frac{d\theta\cos\theta}{r^2}(-\hat{j})$$

$$\vec{E} = \frac{q}{2\pi^2\varepsilon_0 r^2}(-\hat{j})$$



12. A diatomic ideal gas is used in Carnot engine as the working substance. If during the adiabatic expansion part of the cycle, the volume of the gas increases from V to 32 V, the efficiency of the engine is
(1) 0.25 (2) 0.5
(3) 0.75 (4) 0.99

12. (3)

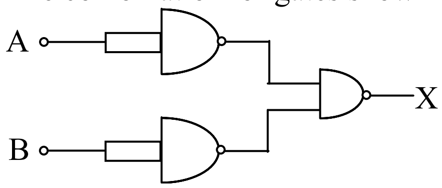
$$\frac{T_1}{T_2} = \left(\frac{32V}{V}\right)^{\frac{7}{5}-1} = 2^2 = 4$$

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{1}{4} = 0.75$$

13. The respective number of significant figures for the numbers 23·023, 0·0003 and $2{\cdot}1 \times 10^{-3}$ are
(1) 4, 4, 2 (2) 5, 1, 2
(3) 5, 1, 5 (4) 5, 5, 2

13. (2)

14. The combination of gates shown below yields



(1) NAND gate
(2) OR gate
(3) NOT gate
(4) XOR gate

14. (2)

$x = \overline{(\overline{A} \cdot \overline{B})} = \overline{\overline{A}} + \overline{\overline{B}} = A + B$

Thus it is OR gate.

15. If a source of power 4 kW produces 10^{20} photons/second, the radiation belongs to a part of the spectrum called

(1) γ-rays
(2) X-rays
(3) ultraviolet rays
(4) microwaves

15. (2)

Wavelength of X-rays ranges from 0.1 – 1 Å

$$\lambda = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8} \times 10^{20}}{4 \times 10^{3}}$$

$\lambda \approx 0.5$ Å

16. A radioactive nucleus (initial mass number A and atomic number Z) emits 3 α-particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be

(1) $\frac{A - Z - 4}{Z - 2}$
(2) $\frac{A - Z - 8}{Z - 4}$
(3) $\frac{A - Z - 4}{Z - 8}$
(4) $\frac{A - Z - 12}{Z - 4}$

16. (3)

Number of protons $= Z - 6 - 2 = Z - 8$

Number of neutrons $= (A - Z) - 6 + 2 = A - Z - 4$

$\therefore$ Ratio $= \frac{A - Z - 4}{Z - 8}$

17. Let there be a spherically symmetric charge distribution with charge density varying as $\rho(r) = \rho_o\left(\frac{5}{4} - \frac{r}{R}\right)$ upto r = R, and $\rho(r) = 0$ for r > R, where r is the distance from the origin. The electric field at a distance r (r < R) from the origin is given by

(1) $\frac{\rho_o r}{3\varepsilon_o}\left(\frac{5}{4} - \frac{r}{R}\right)$
(2) $\frac{4\pi\rho_o r}{3\varepsilon_o}\left(\frac{5}{3} - \frac{r}{R}\right)$
(3) $\frac{\rho_o r}{4\varepsilon_o}\left(\frac{5}{3} - \frac{r}{R}\right)$
(4) $\frac{4\rho_o r}{3\varepsilon_o}\left(\frac{5}{4} - \frac{r}{R}\right)$

17. (3)

Using Gauss's Theorem

$$\int \vec{E} \cdot \overrightarrow{ds} = \frac{1}{\varepsilon_0} q$$

$$E \times 4\pi r^2 = \frac{1}{6\varepsilon_o}\int \rho_o\left(\frac{5}{4}-\frac{r}{R}\right)\times 4\pi r^2 dr$$

on solving

$$E = \frac{\rho_o r}{4\varepsilon_o}\left(\frac{5}{3}-\frac{r}{R}\right)$$

18. In a series LCR circuit R = 200 Ω and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30º. On taking out the inductor form the circuit the current leads the voltage by 30º. The power dissipated in the LCR circuit is

(1) 242 W (2) 305 W
(3) 210 W (4) Zero W

18. (1)

$$<P> = \frac{V_o I_o}{2}\cos\phi$$

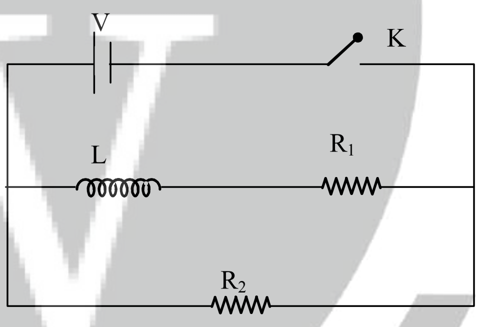
here $\tan 30^o = \frac{X_L}{R}$

also $\tan 30^o = \frac{X_C}{R}$

$\Rightarrow X_L = X_c \Rightarrow Z = R \Rightarrow \phi = 0^o$

$\therefore <P> = \frac{220\times 220}{2\times 200} = 242$ W

19. In the circuit shown below, the key K is closed at t = 0. The current through the battery is



(1) $\frac{V(R_1+R_2)}{R_1R_2}$ at t = 0 and $\frac{V}{R_2}$ at t = ∞

(2) $\frac{VR_1R_2}{\sqrt{R_1^2+R_2^2}}$ at t = 0 and $\frac{V}{R_2}$ at t = ∞

(3) $\frac{V}{R_2}$ at t = 0 and $\frac{V(R_1+R_2)}{R_1R_2}$ at t = ∞ (4) $\frac{V}{R_2}$ at t = 0 and $\frac{VR_1R_2}{\sqrt{R_1^2+R_2^2}}$ at t = ∞

19. (3)

At t = 0 ; $R_L = \infty \Rightarrow i_{t=0} = V/R_2$

At t = ∞ ; $R_L = 0 \Rightarrow i_{t=\infty} = \frac{V(R_1+R_2)}{R_1R_2}$

20. A particle is moving with velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is

(1) $y^2 = x^2$ + constant (2) $y = x^2$ + constant
(3) $y^2 = x$ + constant (4) xy = constant

20. (1)

$\frac{dx}{dt} = Ky$ (1)

$\frac{dy}{dt} = Kx$ (2)

$\frac{dy}{dx} = \frac{x}{y}$

$\int ydy = \int xdx$

$\frac{y^2}{2} = \frac{x^2}{2} + C$

$y^2 = x^2 + \text{constant}$

21. Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then the ratio t_1/t_2 will be

(1) 2 (2) 1

(3) $\frac{1}{2}$ (4) $\frac{1}{4}$

21. (4)

$\frac{q_0}{4} = q_0 e^{-\frac{t_2}{\tau}}$

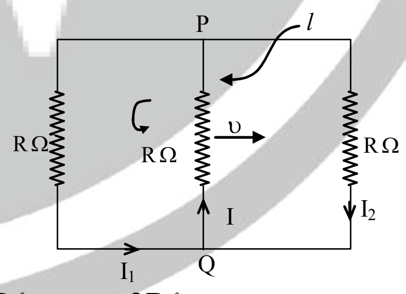
$\Rightarrow \frac{t_2}{\tau} = \ln 4$ (1)

and $\frac{U_0}{2} = U_0 e^{-\frac{2t_1}{\tau}}$

$\Rightarrow \frac{2t_1}{\tau} = \ln 2$ (2)

$\frac{2t_1}{t_2} = \frac{\ln 2}{2\ln 2} = \frac{1}{2} \Rightarrow \frac{t_1}{t_2} = \frac{1}{4}$

22. A rectangular loop has a sliding connector PQ of length l and resistance R Ω and it is moving with a speed υ as shown. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are



(1) $I_1 = I_2 = \frac{B\ell\upsilon}{6R}, I = \frac{B\ell\upsilon}{3R}$ (2) $I_1 = -I_2 = \frac{B\ell\upsilon}{R}, I = \frac{2B\ell\upsilon}{R}$

(3) $I_1 = I_2 = \frac{B\ell\upsilon}{3R}, I = \frac{2B\ell\upsilon}{3R}$ (4) $I_1 = I_2 = I = \frac{B\ell\upsilon}{R}$

22. (3)

$B\ell\upsilon - IR - I_1R = 0$

and $\quad I = I_1 + I_2 = 2I_1$, as $I_1 = I_2$

$\Rightarrow \quad I_1 = I_2 = \dfrac{B\ell v}{3R}$ and $I = \dfrac{2B\ell \upsilon}{3R}$

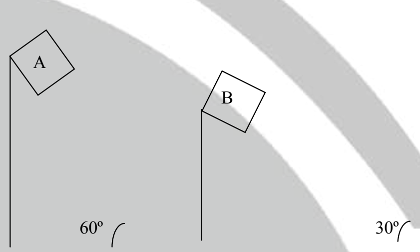
23. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by $y = 0.02(m)\sin\left[2\pi\left(\dfrac{t}{0\cdot04(s)} - \dfrac{x}{0\cdot05(m)}\right)\right]$. The tension in the string is

(1) 6.25 N (2) 4.0 N
(3) 12. 5 N (4) 0. 5 N

23. (1)

$T = \mu v^2 = 0.04 \times \left(\dfrac{25}{2}\right)^2 = 6.25\text{ N}$

24. Two fixed frictionless inclined planes, making an angle 30º and 60º with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B ?



(1) 4.9 ms^{-2} in vertical direction (2) 4.9 ms^{-2} in horizontal direction
(3) 9.8 ms^{-2} in vertical direction (4) Zero

24. (1)

$\vec{a}_{AB} = \vec{a}_A - \vec{a}_B$

$|\vec{a}_{AB}| = (g\sin 60^\circ \cos 30^\circ) - (g\sin 30^\circ \cos 60^\circ)$ in vertical direction

$= 4.9\ \text{m/s}^2$ in vertical direction

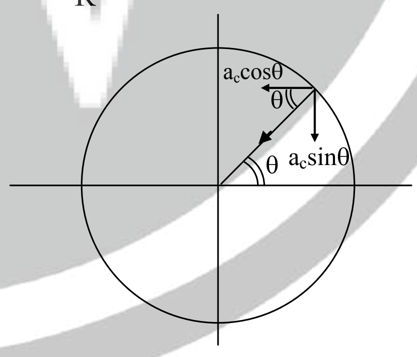
25. For a particle in uniform circular motion, the acceleration $\vec{a}$ at a point P (R, θ) on the circle of radius R is (Here θ is measured from the x-axis)

(1) $\dfrac{\upsilon^2}{R}\hat{i} + \dfrac{\upsilon^2}{R}\hat{j}$ (2) $-\dfrac{\upsilon^2}{R}\cos\theta\,\hat{i} + \dfrac{\upsilon^2}{R}\sin\theta\,\hat{j}$

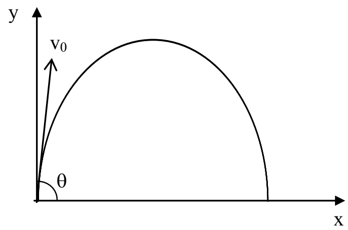
(3) $-\dfrac{\upsilon^2}{R}\sin\theta\,\hat{i} + \dfrac{\upsilon^2}{R}\cos\theta\,\hat{j}$ (4) $-\dfrac{\upsilon^2}{R}\cos\theta\,\hat{i} - \dfrac{\upsilon^2}{R}\sin\theta\,\hat{j}$

25. (4)

$\vec{a} = -\dfrac{\upsilon^2}{R}\cos\theta\,\hat{i} - \dfrac{\upsilon^2}{R}\sin\theta\,\hat{j}$



26. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x – y plane as shown in the figure. At a time $t < \frac{v_0 \sin\theta}{g}$, the angular momentum of the particle is



(1) $\frac{1}{2} mg\, v_0 t^2 \cos\theta\, \hat{i}$ (2) $-mg\, v_0 t^2 \cos\theta\, \hat{j}$

(3) $mg\, v_0 t \cos\theta\, \hat{k}$ (4) $-\frac{1}{2} mg\, v_0 t^2 \cos\theta\, \hat{k}$

where $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axis respectively

26. (4)

$\vec{L} = (\vec{r} \times \vec{p})$

$$\vec{L} = m\left[v_0 \cos\theta\, t\, \hat{i} + \left(v_0 \sin\theta t - \frac{1}{2} g t^2\right)\hat{j}\right] \times \left[v_0 \cos\theta \hat{i} + (v_0 \sin\theta - gt)\hat{j}\right]$$

$\vec{L} = -\frac{1}{2} m g v_0 t^2 \cos\theta\, \hat{k}$

27. Two identical charged spheres are suspended by string of equal lengths. The strings make an angle of 30º with each other. When suspended in a liquid of density 0.8 g cm^{-3}, the angle remains the same. If density of the material of the sphere is 1.6 g cm^{-3}, the dielectric constant of the liquid is

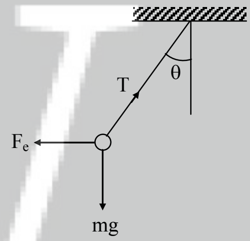
(1) 1 (2) 4

(3) 3 (4) 2

27. (4)

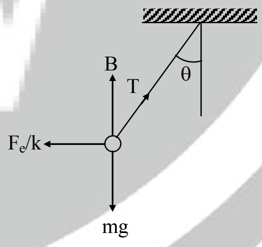
When system is in air

$$\tan\theta = \frac{F_e}{mg} \quad (1)$$



when system is in liquid

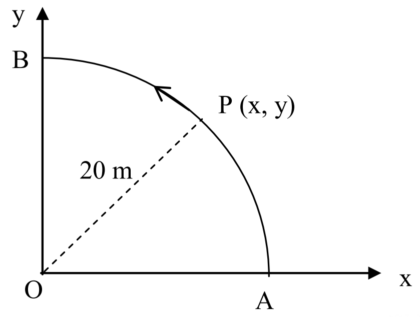
$$\tan\theta = \frac{F_e}{k(mg - B)} \quad (2)$$



on solving equations (1) and (2)

$$k = \frac{1.6}{1.6 - 0.8} = 2$$

28. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of 'P' is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of 'P' when t = 2s is nearly



(1) 14 m/s^2
(2) 13 m/s^2
(3) 12 m/s^2
(4) 7.2 m/s^2

28. (1)

$$a_c = \frac{v^2}{r} = \frac{(3t^2)}{20} = 7.2\,m/s^2 \text{ (at t = 2s)}$$

$$a_t = \frac{dv}{dt} = \frac{d}{dt}(3t^2) = 12\,m/s^2 \text{ (at t = 2s)}$$

$\therefore$ net acceleration

$$a = \sqrt{a_c^2 + a_t^2}$$

$a \approx 14\ m/s^2$

29. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is
$D = [U(x = \infty)] - U_{\text{at equilibrium}}]$, D is

(1) $\frac{b^2}{6a}$
(2) $\frac{b^2}{2a}$
(3) $\frac{b^2}{12a}$
(4) $\frac{b^2}{4a}$

29. (4)

$$U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$$

for equilibrium

$$\frac{dU(x)}{dx} = 0$$

$$\Rightarrow \frac{-12a}{x^{13}} + \frac{6b}{x^7} = 0$$

$$\Rightarrow x^6 = \frac{b}{2a}$$

$$\Rightarrow x = \left(\frac{b}{2a}\right)^{1/6}$$

$\therefore$ U(x) at equilibrium = $\dfrac{a}{\left(\dfrac{b}{2a}\right)^2} - \dfrac{b}{\left(\dfrac{b}{2a}\right)}$

$= \dfrac{b^2}{4a}$

Now D = $\dfrac{b^2}{4a}$ (As U (x = ∞) = 0)

30. Two conductors have the same resistance at 0ºC but their temperature coefficients of resistance are α_1 and α_2. The respective temperature coefficients of their series and parallel combinations are nearly

(1) $\dfrac{\alpha_1+\alpha_2}{2}, \dfrac{\alpha_1+\alpha_2}{2}$

(2) $\dfrac{\alpha_1+\alpha_2}{2}, \alpha_1+\alpha_2$

(3) $\alpha_1+\alpha_2, \dfrac{\alpha_1+\alpha_2}{2}$

(4) $\alpha_1+\alpha_2, \dfrac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}$

30. (1)

$R = R_0 (1 + \alpha t)$

$R_s = R_0 (1 + \alpha_1 t) + R_0 (1 + \alpha_2 t)$

$R_s = 2R_0\left[1+\dfrac{\alpha_1+\alpha_2}{2}t\right]$

$R_s = 2R_0 [1 + \alpha_s t]$

comparing $\alpha_s = \dfrac{\alpha_1+\alpha_2}{2}$

$R_p = \dfrac{R_1R_2}{R_1R_2} = \dfrac{R_o^2(1+(\alpha_1+\alpha_2)t)}{2R_0\left[1+\dfrac{\alpha_1+\alpha_2}{2}\right]t}$

$\therefore \alpha_p = \dfrac{\alpha_1+\alpha_2}{2}$

PART B – CHEMISTRY

31. In aqueous solution the ionization constants for carbonic acid are
$K_1 = 4.2 \times 10^{-7}$ and $K_2 = 4.8 \times 10^{-11}$
Select the correct statement for a saturated 0.034 M solution of the carbonic acid.
(1) The concentration of H^+ is double that of CO_3^{2-}.
(2) The concentration of CO_3^{2-} is 0.034 M.
(3) The concentration of CO_3^{2-} is greater than that of HCO_3^-
(4) The concentrations of H^+ and HCO_3^- are approximately equal.

31. **(4)**
$K_1 = 4.2 \times 10^{-7}$, $K_2 = 4.8 \times 10^{-11}$
As $K_1 >> K_2$ so nearly all of the H^+ ions comes form first dissociation step.
$H_2CO_3 \rightleftharpoons H^+ + HCO_3^-$
$\therefore\ [H^+] \approx [HCO_3^-]$

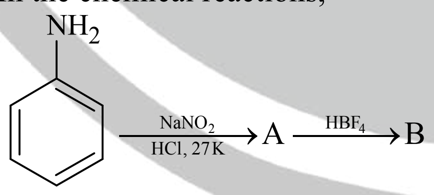
32. Solubility product of silver bromide is 5.0×10^{-13}. The quantity of potassium bromide (molar mass taken as 120 g mol^{-1}) to be added to 1 litre of 0.05 M solution of silver nitrate to start the precipitation of AgBr is
(1) 5.0×10^{-8} g
(2) 1.2×10^{-10} g
(3) 1.2×10^{-9} g
(4) 6.2×10^{-5} g

32. **(3)**
$k_{sp_{AgBr}} = 5 \times 10^{-13}$
$k_{sp} = [Ag^+][Br^-]$
$$\therefore\ [Br^-] = \frac{k_{sp}}{[Ag^+]} = \frac{5 \times 10^{-13}}{5 \times 10^{-2}} = 10^{-11}$$
$$\therefore\ 10^{-11} = \frac{w}{m} \text{ or } w = 120 \times 10^{-11} = 1.2 \times 10^{-9} g$$

33. the correct sequence which shows decreasing order of the ionic radii of the elements is
(1) $O^{2-} > F^- > Na^+ > Mg^{2+} > Al^{3+}$
(2) $Al^{3+} > Mg^{2+} > Na^+ > F^- > O^{2-}$
(3) $Na^+ > Mg^{2+} > Al^{3+} > O^{2-} > F^-$
(4) $Na^+ > F^- > Mg^{2+} > O^{2-} > Al^{3+}$

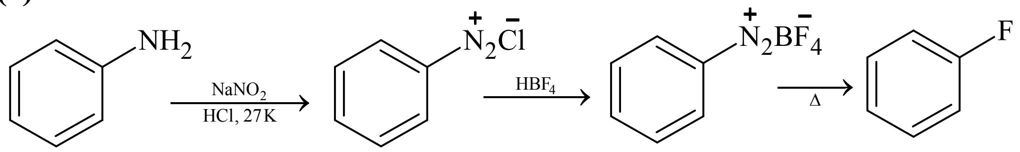
33. **(1)**
Among isoelectronics $r \propto \frac{1}{Z}$

34. In the chemical reactions,



the compounds 'A' and 'B' respectively are
(1) nitrobenzene and chlorobenzene
(2) nitrobenzene and fluorobenzene
(3) phenol and benzene
(4) benzene diazonium chloride and fluorobenzene

34. **(4)**



35. If 10^{-4} dm^3 of water is introduce into a 1.0 dm^3 flask at 300 K, how many moles of water are in the vapour phase when equilibrium is established?
(Given : Vapour pressure of H_2O at 300 K is 3170 Pa; R = 8.314 J K^{-1} mol^{-1})

(1) 1.27×10^{-3} mol (2) 5.56×10^{-3} mol
(3) 1.53×10^{-2} mol (4) 4.46×10^{-2} mol

35. **(2)**

$n_{H_2O_2} = ?\quad P^o_{H_2O} = 3170 \text{ Pa}\quad R = 8.314$

$n = \frac{PV}{RT} = \frac{3170 \times 1}{8.314 \times 300} = 1.27$

$d_{H_2O} = 10^{-4} \text{kg} = \frac{10^{-1} \text{g}}{18} = 5.56 \times 10^{-3} \text{mol}$

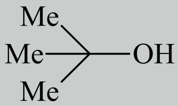
Because to develop 3170 Pa 1.27 moles are required but the actual amount is less so complete vaporisation occurs.

36. From amongst the following alcohols the one that would react fastest with conc. HCl and anhydrous $ZnCl_2$, is

(1) 1-Butanol (2) 2-Butanol
(3) 2-Methylpropan-2-ol (4) 2-Methylpropanol

36. **(3)**

This is Luca's test and 3° alcohol responds quickly.

∴ 

2-methyl Propan-2-ol

37. If sodium sulphate is considered to be completely dissociated into cations and anions in aqueous solution, the change in freezing point of water (ΔT_f), when 0.01 mol of sodium sulphate is dissolved in 1 kg of water, is (K_f = 1.86 K kg mol^{-1})

(1) 0.0186 K (2) 0.0372 K
(3) 0.0558 K (4) 0.0744 K

37. **(3)**

$\Delta T_f = i\, k_f\, m = 3 \times 1.86 \times 0.01$
$= 0.0558$ K

38. Three reactions involving $H_2PO_4^-$ are given below:

i) $H_3PO_4 + H_2O \rightarrow H_3O^+ + H_2PO_4^-$
ii) $H_2PO_4^- + H_2O \rightarrow HPO_4^{2-} + H_3O^+$
iii) $H_2PO_4^- + OH^- \rightarrow H_3PO_4 + O^{2-}$

In which of the above does $H_2PO_4^-$ act as an acid?

(1) (i) only (2) (ii) only
(3) (i) and (ii) (4) (iii) only

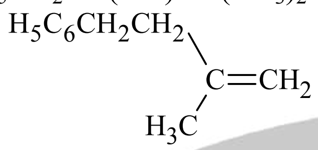
38. **(2)**

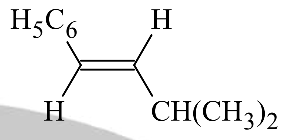
$H_2PO_4^- + H_2O \longrightarrow HPO_4^{2-} + H_3O^+$

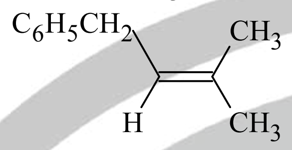
In the above reaction $H_2PO_4^-$ as acid

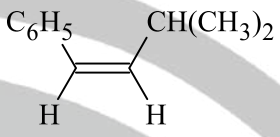
39. The main product of the following reaction is

$C_6H_5CH_2CH(OH)CH(CH_3)_2 \xrightarrow{\text{conc. } H_2SO_4} ?$

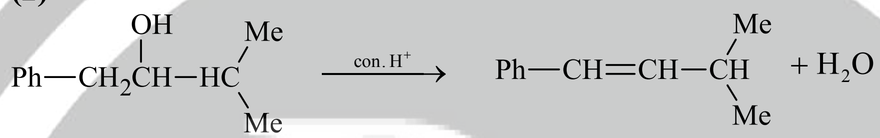
(1) 

(2) 

(3) 

(4) 

39. **(2)**



40. The energy required to break one mole of Cl – Cl bonds in Cl_2 is 242 kJ mol^{-1}. The longest wavelength of light capable of breaking a single Cl – Cl bond is

($c = 3 \times 10^8\ ms^{-1}$ and $N_A = 6.02 \times 10^{23}\ mol^{-1}$)

(1) 494 nm
(2) 594 nm
(3) 640 nm
(4) 700 nm

40. **(1)**

$E_{Cl-Cl} = 742$ kJ/mol

$$E/\text{molecule} = \frac{242}{N_{av}} = \frac{hc}{\lambda}$$

$$\therefore\ \lambda = \frac{hc\, N_{av}}{242}$$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^8 \times 6.02 \times 10^{23}}{242 \times 10^3} = 0.49 \times 10^{-6} = 494 \text{ nm}$$

41. 29.5 mg of an organic compound containing nitrogen was digested according to Kjeldahl's method and the evolved ammonia was absorbed in 20 mL of 0.1 M HCl solution. The excess of the acid required 15 mL of 0.1 M NaOH solution for complete neutralization. The percentage of nitrogen in the compound is

(1) 29.5
(2) 59.0
(3) 47.4
(4) 23.7

41. **(4)**

Meq of NH_3 absorbed by acid = $20 \times 0.1 - 15 \times 0.1 = 0.5$

moles of $NH_3 = 5 \times 10^{-4}$

$\therefore$ wt of N = $14 \times 5 \times 10^{-4}$ gm

$\therefore$ w = $70 \times 10^{-4} = 7.0$ mg

$$\% \text{ of N} = \frac{7.0}{29.5} \times 100 = 23.72\%$$

42. Ionisation energy of He^+ is 19.6×10^{-18} J $atom^{-1}$. The energy of the first stationary state (n = 1) of Li^{2+} is

(1) 8.82×10^{-17} J $atom^{-1}$
(2) 4.41×10^{-16} J $atom^{-1}$
(3) -4.41×10^{-17} J $atom^{-1}$
(4) -2.2×10^{-15} J $atom^{-1}$

42. **(3)**

I_p of $He^+ = 19.6 \times 10^{-18}$ J atm^{-1}

$$19.6\times10^{-18} = \frac{Z^2}{n^2} \,.\, k$$

$$\therefore\ k = 19.6 \times 10^{-18} \times \frac{n^2}{Z^2} = 19.6\times10^{-18} \times \frac{1}{4}$$

$$E = \frac{-19.6\times10^{-18}}{4} \cdot \frac{Z^2}{n^2} = \frac{-19.6\times10^{-18}}{4} \times \frac{9}{1}$$

$= -44.1 \times 10^{-18} = -4.41 \times 10^{-17}$ J atm^{-1}

43. On mixing, heptane and octane form an ideal solution. At 373 K, the vapour pressures of the two liquid components (heptane and octane) are 105 kPa and 45 kPa respectively. Vapour pressure of the solution obtained by mixing 25.0g of heptane and 35g of octane will be (molar mass of heptane = 100 g mol^{-1} and of octane = 114g mol^{-1}).

(1) 144.5 kPa
(2) 72.0 kPa
(3) 36.1 kPa
(4) 96.2 kPa

43. **(2)**

$P_A^o = 105 kPa \quad P_B^o = 45 kPa$

$$\text{total moles} = \frac{25g}{100} + \frac{35g}{114}$$

$= 0.25 + 0.31 = 0.56$

$$\therefore\ x_A = \frac{0.25}{0.56} \quad x_B = \frac{0.31}{0.56}$$

$$P = 105 \times \frac{25}{56} + 45 \times \frac{31}{56} = \frac{2625 + 1395}{56} = \frac{4020}{56} = 71.78 \approx 72 \text{ kPa}$$

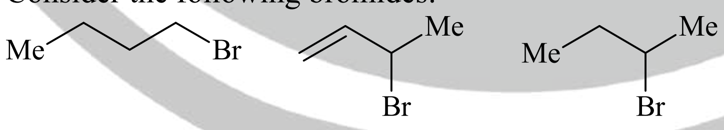
44. Which one of the following has an optical isomer?

(1) $[Zn(en)_2]^{2+}$
(2) $[Zn(en)(NH_3)_2]^{2+}$
(3) $[Co(en)_3]^{3+}$
(4) $[Co(H_2O)_4(en)]^{3+}$

(en = ethylenediamine)

44. **(3)**

45. Consider the following bromides:

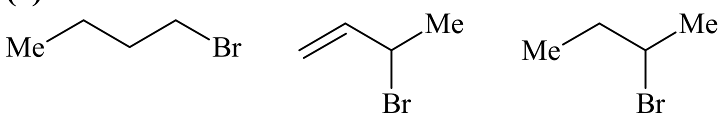


(A) (B) (C)

The correct order of S_N1 reactivity is

(1) A > B > C
(2) B > C > A
(3) B > A > C
(4) C > B > A

45. **(2)**



(A) (B) (C)

$\therefore$ B > C > A as carbocation formed from B is 2° allylic.

46. One mole of a symmetrical alkene on ozonolysis gives two moles of an aldehyde having a molecular mass of 44u. The alkene is

(1) ethene (2) propene

(3) 1-butene (4) 2-butene

46. **(4)**

$$CH_3CH = \boxed{O\ +\ O} = CH - CH_3 \Rightarrow \underset{\text{But}-2-\text{ene}}{CH_3 - CH = CH - CH_3}$$

47. Consider the reaction:

$Cl_2(aq) + H_2S(aq) \rightarrow S(s) + 2H^+ (aq) + 2Cl^- (aq)$

The rate equation for this reaction is

rate = $k[Cl_2]\ [H_2S]$

Which of these mechanisms is/are consistent with this rate equation?

A. $Cl_2 + H_2S \rightarrow H^+ + Cl^- + Cl^+ + HS^-$ (slow)

$Cl^+ + HS^- \rightarrow H^+ + Cl^- + S$ (fast)

B. $H_2S \Leftrightarrow H^+ + HS^-$ (fast equilibrium)

$Cl_2 + HS^- \rightarrow 2\ Cl^- + H^+ + S$ (slow)

(1) A only (2) B only

(3) Both A and B (4) Neither A nor B

47. **(1)**

$r = k\ [Cl_2][H_2S]$

As 1st step is slowest so route A is valid

and for route B

$r = k\ [Cl_2][HS^-]$ $\quad k_c = \dfrac{[H^+][HS^-]}{[H_2S]}$

$$r = kk_c \frac{[H_2S]}{[H^+]}[Cl_2]$$

This is not desired rate law because $[H^+]$ is not present in the original reaction so route B is not valid.

48. The Gibbs energy for the decomposition of Al_2O_3 at 500 °C is as follows:

$\frac{2}{3}Al_2O_3 \rightarrow \frac{4}{3}Al + O_2$, $\Delta_rG = +\ 966\ kJ\ mol^{-1}$. The potential difference needed for electrolytic reduction of Al_2O_3 at 500°C is at least

(1) 5.0 V (2) 4.5V

(3) 3.0 V (4) 2.5 V

48. **(4)**

$\Delta G = 966$ for 2/3 mol of Al_2O_3

$\therefore$ for 1 mol of $Al_2O_3 = \dfrac{966 \times 3}{2}$

Now $\Delta G = -nFE$

$$\therefore \quad E = -\frac{\Delta G}{nF} = \frac{966 \times 3 \times 1000}{2 \times 6 \times 96500} J/c$$

$$\therefore \quad E = \frac{10}{4} = 2.5V$$

49. The correct order of increasing basicity of the given conjugate bases ($R = CH_3$) is

(1) $RCO\overline{O} < HC \equiv \overline{C} < \overline{N}H_2 < \overline{R}$
(2) $RCO\overline{O} < HC \equiv \overline{C} < \overline{R} < \overline{N}H_2$
(3) $\overline{R} < HC \equiv \overline{C} < RCO\overline{O} < \overline{N}H_2$
(4) $RCO\overline{O} < \overline{N}H_2 < HC \equiv \overline{C} < \overline{R}$

49. **(1)**

$RCO\overline{O} < HC \equiv \overline{C} < \overline{N}H_2 < \overline{R}$

50. The edge length of a face centered cubic cell of an ionic substance is 508 pm. If the radius of the cation is 110 pm, the radius of the anion is

(1) 144 pm
(2) 288 pm
(3) 398 pm
(4) 618 pm

50. **(1)**

$a = 508$ pm $\quad r^+ = 110$ pm $\quad$ fcc

Assuming NaCl type lattice

The anion is present in fcc lattice and cation is present in octahedral hole

$\therefore \quad a = 2(r^+ + r^-)$

$\therefore \quad 508 = 220 + 2r^-$

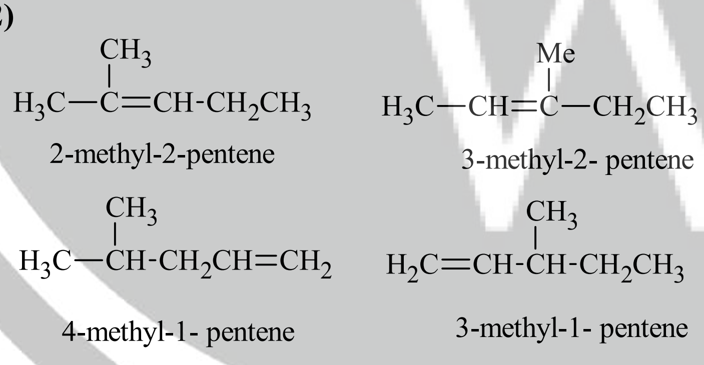
$$\therefore \quad r^- = \frac{508 - 220}{2} = 144 \text{ pm}$$

But the question is incomplete as lattice type is not given.

51. Out of the following the alkene that exhibits optical isomerism is

(1) 2-methyl-2-pentene
(2) 3-methyl-2-pentene
(3) 4-methyl-1-pentene
(4) 3-methyl-1-pentene

51. **(2)**



52. For a particular reversible reaction at temperature T, ΔH and ΔS were found to be both +ve. If T_e is the temperature at equilibrium, the reaction would be spontaneous when

(1) $T = T_e$
(2) $T_e > T$
(3) $T > T_e$
(4) T_e is 5 times T

52. **(3)**

$\Delta H = +ve \quad \Delta S = +ve. \quad T_e$ = equilibrium temperature

At equilibrium $\Delta H = T\Delta S$ as $\Delta G = 0$

as $T > T_e$ the ΔG become –ve and reaction achieves spontaneity.

53. Percentages of free space in cubic close packed structure and in body centred packed structure are respectively

(1) 48% and 25%
(2) 30% and 25%
(3) 26% and 32%
(4) 32% and 48%

53. **(3)**

Free space in Fcc = 26%
Free space in Bcc = 32%

54. The polymer containing strong intermolecular forces e.g. hydrogen bonding is

(1) natural rubber
(2) Teflon
(3) nylon 6, 6
(4) polystyrene

54. **(3)**

55. At 25°C, the solubility product of $Mg(OH)_2$ is 1.0×10^{-11}. At which pH, will Mg^{2+} ions start precipitating in the form of $Mg(OH)_2$ from a solution of 0.001 M Mg^{2+} ions?

(1) 8
(2) 9
(3) 10
(4) 11

55. **(3)**

$$k_{sp} Mg(OH)_2 = 10^{-11}$$

$$10^{-11} = [Mg^{2+}][OH^-]^2$$

$$\therefore \quad [OH^-] = \left(\frac{10^{-11}}{10^{-3}}\right)^{\frac{1}{2}} = 10^{-4}$$

$$\therefore \quad pH = 10$$

56. The correct order of $E^o_{M^{2+}/M}$ values with negative sign for the four successive elements Cr, Mn, Fe and Co is

(1) Cr > Mn > Fe > Co
(2) Mn > Cr > Fe > Co
(3) Cr > Fe > Mn > Co
(4) Fe > Mn > Cr > Co

56. **(2)**

Mn > Cr > Fe > Co

57. Biuret test is not given by

(1) proteins
(2) carbohydrates
(3) polypeptides
(4) urea

57. **(2)**

Proteins, Polypeptides and Urea all three have peptide linkage under Biuret test condition.

58. The time for half life period of a certain reaction A ⟶ Products is 1 hour. When the initial concentration of the reactant 'A', is 2.0 mol L^{-1}, how much time does it take for its concentration to come from 0.50 to 0.25 mol L^{-1} if it is a zero order reaction?

(1) 1 h
(2) 4 h
(3) 0.5 h
(4) 0.25 h

58. **(4)**

$t_{1/2} = 1$ hr $[A]_0 = 2M$

$$k = \frac{x}{t} = \frac{[A]_0 - [A]}{t}$$

$k = \frac{2-1}{1} = 1$ mol $lit^{-1}h^{-1}$

again $1 = \frac{0.50 - 0.25}{t}$

t = 0.25 hr.

59. A solution containing 2.675 g of $CoCl_3$. 6 NH_3 (molar mass = 267 .5g mol^{-1}) is passed through a cation exchanger. The chloride ions obtained in solution were treated with excess of $AgNO_3$ to give 4.78 g of AgCl (molar mass = 143.5 g mol^{-1}). The formula of the complex is (At. mass of Ag = 108 u)

(1) $[CoCl(NH_3)_5]Cl_2$
(2) $[Co(NH_3)_6]Cl_3$
(3) $[CoCl_2(NH_3)_4]Cl$
(4) $[CoCl_3(NH_3)_3]$

59. **(2)**

$$2.675g\ CoCl_3 .6NH_3\ (267.5) \xrightarrow{AgNO_3\ excess} 4.78g\ AgCl\ (143.5)$$

moles of AgCl = $\frac{4.78}{143.5} = 0.033$ mole

moles of complex = $\frac{2.675}{267.5} = 0.01$ moles

So, 0.01 mol $\longrightarrow$ 0.033 mol Cl^-

1 mol = $\frac{.033}{.01} = 3.3$ mol Cl^-

$\therefore$ All 3 Cl^- were outside the coordination sphere.

60. The standard enthalpy of formation of NH_3 is –46.0 kJ mol^{-1}. If the enthalpy of formation of H_2 from its atoms is –436 kJ mol^{-1} and that of N_2 is –712 kJ mol^{-1}, the average bond enthalpy of N – H bond in NH_3 is

(1) –1102 kJ mol^{-1}
(2) –964 kJ mol^{-1}
(3) + 352 kJ mol^{-1}
(4) +1056 kJ mol^{-1}

60. **(3)**

$N_2 \rightarrow 2N \quad \Delta H_1 = +712$

$3H_2 \rightarrow 6H \quad \Delta H_2 = +3 \times 436$

$2N + 6H \rightarrow 2NH_3 \quad \Delta H_3$

$N_2 + 3H_2 \rightarrow 2NH_3 \quad \Delta H_4 = -2 \times 46$

$-2 \times 46 = +712 + 3 \times 436 + \Delta H_3$

$-92 = +712 + 1308 + \Delta H_3$

$\Delta H_3 = -92 - 712 - 1308$

$= -2112$

$E_{N-H} = -\frac{2112}{6} = -352$

So the average bond enthalpy of N-H bond in NH_3 is 352 kj/mol.

PART C – MATHEMATICS

61. Consider the following relations :

R = {(x, y) | x, y are real numbers and x = wy for some rational number w};

S = $\{(\frac{m}{n}, \frac{p}{q})$ |m, n, p and q are integers such that n, q $\neq$ 0 and qm = pn}.

Then

(1) R is an equivalence relation but S is not an equivalence relation

(2) neither R nor S is an equivalence relation

(3) S is an equivalence relation but R is not an equivalence relation

(4) R and S both are equivalence relations

61. (3)

As $(0, 1) \in R$ but $(1, 0) \notin R$

Hence R is not symmetric

$\Rightarrow$ R is not equivalence relation

As $\left(\frac{m}{n}, \frac{m}{n}\right) \in S$ and $\left(\frac{m}{n}, \frac{p}{q}\right) \in S \Rightarrow \left(\frac{p}{q}, \frac{m}{n}\right) \in S$

and if $\left(\frac{m}{n}, \frac{p}{q}\right), \left(\frac{p}{q}, \frac{r}{s}\right) \in S$

$\Rightarrow$ mq = np and ps = qr

$\Rightarrow$ ms = nr $\Rightarrow \left(\frac{m}{n}, \frac{r}{s}\right) \in S$.

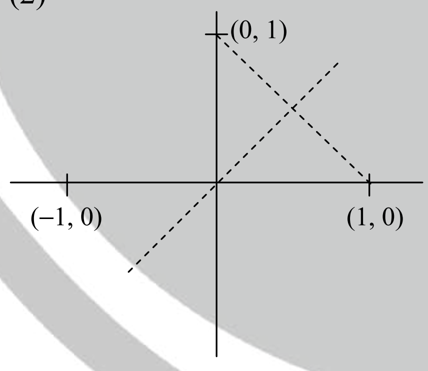
Hence S is equivalence relation

62. The number of complex numbers z such that $|z - 1| = |z + 1| = |z - i|$ equals

(1) 0 (2) 1

(3) 2 (4) ∞

62. (2)



63. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$

(1) −2 (2) −1

(3) 1 (4) 2

63. (3)

$(\alpha, \beta) \equiv (-\omega, -\omega^2)$

$\Rightarrow (-\omega)^{2009} + (-\omega^2)^{2009} = -(\omega^2 + \omega^4) = 1$

64. Consider the system of linear equations

$x_1 + 2x_2 + x_3 = 3$

$2x_1 + 3x_2 + x_3 = 3$

$3x_1 + 5x_2 + 2x_3 = 1$

The system has

(1) infinite number of solutions (2) exactly 3 solutions

(3) a unique solution (4) no solutions

64. (4)

$$\begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 5 & 2 \end{vmatrix} = 0, \begin{vmatrix} 3 & 2 & 1 \\ 3 & 3 & 1 \\ 1 & 5 & 2 \end{vmatrix} = 5$$

65. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is

(1) 3 (2) 36

(3) 66 (4) 108

65. (4)

${}^3C_2 \cdot {}^9C_2 = 108$

66. Let $f : (-1, 1) \to R$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$. Let $g(x) = [f(2f(x) + 2]^2$. Then $g'(0) =$

(1) 4 (2) −4

(3) 0 (4) −2

66. (2)

$g'(x) = 2f(2f(x) + 2) \times (f'(2f(x) + 2)\,(2f'(x))$

$g'(0) = 4f(0)\,(f'(0))^2 = -4.$

67. Let $f : R \to R$ be a positive increasing function with $\lim\limits_{x\to\infty} \frac{f(3x)}{f(x)} = 1.$

Then $\lim\limits_{x\to\infty} \frac{f(2x)}{f(x)} =$

(1) 1 (2) $\frac{2}{3}$

(3) $\frac{3}{2}$ (4) 3

67. (1)

AS f(x) is increasing and positive

$$\Rightarrow \frac{f(x)}{f(x)} < \frac{f(2x)}{f(x)} < \frac{f(3x)}{f(x)} \Rightarrow \lim_{x\to\infty} 1 \le \lim_{x\to\infty} \frac{f(2x)}{f(x)} \le \lim_{x\to\infty} \frac{f(x)}{f(x)}$$

$\Rightarrow \lim_{x\to\infty}\frac{f(2x)}{f(x)}=1$

68. Let p(x) be a function defined on R such that p′(x) = p′(1 − x), for all x ∈ [0, 1], p(0) = 1 and p(1) = 41. Then $\int_0^1 p(x)dx$ equals

(1) $\sqrt{41}$ (2) 21
(3) 41 (4) 42

68. (2)

$\Rightarrow$ p′(x) = p′(1 – x) $\Rightarrow$ p(x) + P(1 – x) = C

p(0) + p(1) = C = 42

$\Rightarrow \int_0^1 p(x)dx = \int_0^{1/2}\left(p(x)+p(1-x)\right)dx$

$= 42\int_0^{1/2} dx = 21$

69. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = \ldots\ldots a_{10} = 150$ and $a_{10}, a_{11}, \ldots\ldots$ are in an AP with common difference −2, then the time taken by him to count all notes is

(1) 24 minutes (2) 34 minutes
(3) 125 minutes (4) 135 minutes

69. (2)

By 10^{th} second he has counted 1500 currency notes and let remaining are counted in further k minutes

$\Rightarrow \frac{k}{2}\left(2\times148+(k-1)(-2)\right)=3000$

k(149– k) = 3000

$\Rightarrow$ k = 24

Total time taken = 10 + 24 = 34

70. The equation of the tangent to the curve $y = x+\frac{4}{x^2}$, that is parallel to the x–axis, is

(1) y = 0 (2) y = 1
(3) y = 2 (4) y = 3

70. (4)

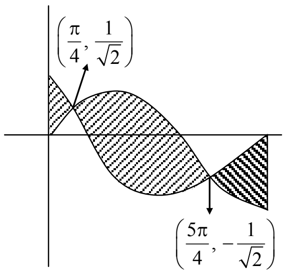
$y' = 1-\frac{8}{x^3}=0 \quad x^3 = 8 \Rightarrow x = 2$

$\Rightarrow$ tangent is y = 3

71. The area bounded y the curves y = cos x and y = sinx between the ordinates x = 0 and $x=\frac{3\pi}{2}$ is

(1) $4\sqrt{2}-2$ (2) $4\sqrt{2}+2$
(3) $4\sqrt{2}-1$ (4) $4\sqrt{2}+1$

71. (1)



$\Rightarrow$

$$\int_0^{\pi/4}(\cos x-\sin x)dx+\int_{\pi/4}^{5\pi/4}(\sin x-\cos x)dx+\int_{5\pi/4}^{3\pi/2}(\cos x-\sin x)dx$$

$$= 4\sqrt{2}-2$$

72. Solution of the differential equation cos x dy = y(sin x – y) dx, $0 < x < \frac{\pi}{2}$ is

(1) sec x = (tan x + c)y
(2) y sec x = tan x + c
(3) y tan x = sec x + c
(4) tan x = (sec x + c) y

72. (1)

$$\frac{\cos x}{y^2}\frac{dy}{dx}-\frac{\sin x}{y}=-1$$

$\Rightarrow \frac{1}{y^2}\frac{dy}{dx}-\frac{\tan x}{y}=-\sec x$, Let $\frac{1}{y}=t$

$\Rightarrow \frac{dt}{dx}+\tan x\, t=\sec x$

$\Rightarrow \frac{\sec x}{y}=\int \sec^2 x dx=\tan x+c$

$\Rightarrow$ secx = (tanx + c) y

73. Let $\vec{a}=\hat{j}-\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}-\hat{k}$. Then the vector $\vec{b}$ satisfying $\vec{a}\times\vec{b}+\vec{c}=\vec{0}$ and $\vec{a}\cdot\vec{b}=3$ is

(1) $-\hat{i}+\hat{j}-2\hat{k}$
(2) $2\hat{i}-\hat{j}+2\hat{k}$
(3) $\hat{i}-j-2\hat{k}$
(4) $\hat{i}+\hat{j}-2\hat{k}$

73. (1)

$\vec{a}\times\left(\vec{a}\times\vec{b}+\vec{c}\right)=0$

$\Rightarrow \left(\vec{a}.\vec{b}\right)\vec{a}-\vec{a}.\vec{a}\vec{b}+\vec{a}\times\vec{c}=0$

$\Rightarrow 3\left(\hat{j}-\hat{k}\right)-2(\vec{b})-2\hat{i}-\hat{j}-\hat{k}=0$

$\Rightarrow \vec{b}=-\hat{i}+\hat{j}-2\hat{k}$

74. If the vectors $\vec{a}=\hat{i}-\hat{j}+2\hat{k}$, $\vec{b}=2\hat{i}+4\hat{j}+\hat{k}$ and $\vec{c}=\lambda\hat{i}+\hat{j}+\mu\hat{k}$ are mutually orthogonal, then (λ, μ) =

(1) (–3, 2)
(2) (2, –3)
(3) (–2, 3)
(4) (3, –2)

74. (1)

$\vec{b}.\vec{c}=0$, $\vec{a}.\vec{c}=0$

2λ + μ = –4, λ + 2μ = 1

λ = –3, μ = 2.

75. If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of P is

(1) $x = 1$ (2) $2x + 1 = 0$

(3) $x = -1$ (4) $2x - 1 = 0$

75. (3)

Locus is directrix, i.e., $x = -1$.

76. The line L given by $\frac{x}{5}+\frac{y}{b}=1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c}+\frac{y}{3}=1$. Then the distance between L and K is

(1) $\frac{23}{\sqrt{15}}$ (2) $\sqrt{17}$

(3) $\frac{17}{\sqrt{15}}$ (4) $\frac{23}{\sqrt{17}}$

76. (4)

$\frac{13}{5}+\frac{32}{b}=1 \Rightarrow b = -20$, also $\frac{5}{c} = -\frac{b}{3}$

$\Rightarrow c = -\frac{3}{4}$

$\Rightarrow$ both lines are $4x - y = 20$, $4x - y = -3$

perpendicular between line $\frac{23}{\sqrt{17}}$

77. A line AB in three-dimensional space makes angles 45° and 120° with the positive x–axis and the positive y–axis respectively. If AB makes an acute angle θ with the positive z–axis, then θ equals

(1) 30° (2) 45°

(3) 60° (4) 75°

77. (3)

$\cos^2 45 + \cos^2 120 + \cos^2\theta = 1$

$\cos^2\theta = \frac{1}{4} \Rightarrow \cos\theta = \frac{1}{2}, \theta = 60^o$.

78. Let S be a non–empty subset of R. Consider the following statement :

P : There is a rational number $x \in S$ such that $x > 0$.

Which of the following statements is the negation of the statement P ?

(1) There is a rational number $x \in S$ such that $x \le 0$.

(2) There is no rational number $x \in S$ such that $x \le 0$.

(3) Every rational number $x \in S$ satisfies $x \le 0$.

(4) $x \in S$ and $x \le 0 \Rightarrow x$ is not rational.

78. (3)

79. Let $(\alpha+\beta)=\frac{4}{5}$ and let $\sin(\alpha-\beta)=\frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then $\tan 2\alpha =$

(1) $\frac{25}{16}$
(2) $\frac{56}{33}$
(3) $\frac{19}{12}$
(4) $\frac{20}{7}$

79. (2)

$\cos(\alpha + \beta) = \frac{4}{5}$, $\tan(\alpha + \beta) = \frac{3}{4}$

$\sin(\alpha - \beta) = \frac{5}{13}$, $\tan(\alpha - \beta) = \frac{5}{12}$

$$\tan(\alpha+\beta+\alpha-\beta)=\frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$$

$$=\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{15}{48}}=\frac{14\times4}{33}=\frac{56}{33}$$

80. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if

(1) $-85 < m < -35$
(2) $-35 < m < 15$
(3) $15 < m < 65$
(4) $35 < m < 85$

80. (2)

$(x - 2)^2 + (y - 4)^2 = 5^2$

$\Rightarrow \frac{|6-16-m|}{5} < 5$

$\Rightarrow |m + 10| < 25$

$-35 < m < 15$

81. For two data sets, each of the size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is

(1) $\frac{5}{2}$
(2) $\frac{11}{2}$
(3) 6
(4) $\frac{13}{2}$

81. (2)

Combined mean $(\bar{x}) = \frac{n_1\bar{x}_1+\bar{x}_2n_2}{n_1+n_2}=\frac{5\times2+5\times4}{10}=3$

Variance $= \frac{1}{n_1+n_2}\left(n_1(\sigma_1^2+(\bar{x}_1-\bar{x})^2+n_2\left(\sigma_2^2+(\bar{x}_2-\bar{x})^2\right)\right)$

$\frac{5}{10}(4+1+5+1)=\frac{11}{2}$

82. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours is

(1) $\frac{1}{3}$ (2) $\frac{2}{7}$

(3) $\frac{1}{21}$ (4) $\frac{2}{23}$

82. (2)

Required probability $= \frac{{}^3C_1 \times {}^4C_1 \times {}^2C_1}{{}^9C_3} = \frac{2}{7}$.

83. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A ***false*** statement among the following is

(1) There is a regular polygon with $\frac{r}{R} = \frac{1}{2}$ (2) There is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$

(3) There is a regular polygon with $\frac{r}{R} = \frac{2}{3}$ (4) There is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$

83. (3)

If regular polygon is n–sided and circum radius is R

$\Rightarrow r = R\cos\frac{\pi}{n}$

$\Rightarrow \frac{r}{R} = \cos\frac{\pi}{n} \neq \frac{2}{3}$ (as n is integer)

84. The number of 3 × 3 non–singular matrices, with four entries as 1 and all other entries as 0, is

(1) less than 4 (2) 5

(3) 6 (4) at least 7

84. (4)

$A = \begin{bmatrix} 1 & - & - \\ - & 1 & - \\ - & - & 1 \end{bmatrix}$ if all the elements of leading diagonal are 1, then its determinants is non zero and 6 such matrices are possible.

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ is also an matrix which is non singular.

85. Let f : R → R be defined by

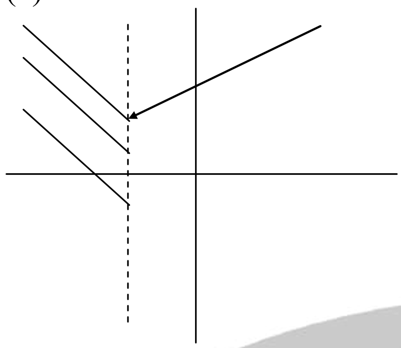
$$f(x) = \begin{cases} k - 2x, & \text{if } x \le -1 \\ 2x + 3, & \text{if } x > -1 \end{cases}$$

If f has a local minimum at x = −1, then a possible value of k is

(1) 1 (2) 0

(3) $-\frac{1}{2}$ (4) −1

85. (4)



For minimum at $x = -1$

$k + 2 \le 1$

$k \le -1$

Directions : Questions number 86 to 90 on Assertion – Reason type questions. Each of these questions contains two statements.

Statement–1 : (Assertion) and

Statement–2 : (Reasons).

Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

86. Four numbers are chosen at random (without replacement) from the set {1, 2, 3, …., 20}

Statement–1 : The probability that the chosen numbers when arranged in same order will form an AP is $\frac{1}{85}$.

Statement–2 : If the four chosen numbers from an AP, then the set of all possible values of common difference is $\{\pm1, \pm2, \pm3, \pm4, \pm5\}$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(C) Statement-1 is True, Statement-2 is False.

(D) Statement-1 is False, Statement-2 is True.

86. (3)

Second statement is wrong as common difference can be 6.

i.e., {1, 7, 13, 19}

87. Let $S_1 = \sum_{j=1}^{10} j(j-1)\,{}^{10}C_j$, $S_2 = \sum_{j=1}^{10} j\,{}^{10}C_j$ and $S_3 = \sum_{j=1}^{10} j^2\,{}^{10}C_j$.

Statement–1 : $S_3 = 55 \times 2^9$.

Statement–2 : $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(C) Statement-1 is True, Statement-2 is False.

(D) Statement-1 is False, Statement-2 is True.

87. (3)

$$\sum_{j=1}^{10} j\,{}^{10}C_j = \sum_{j=1}^{10} 10\,{}^{9}C_{j-1} = 10.2^9$$

Second statement is wrong.

88. **Statement–1 :** The point A(3, 1, 6) is the mirror image or the point B(1, 3, 4) in the plane $x - y + z = 5$.

Statement–2 : The plane $x - y + z = 5$ bisects the line segment joining A(3, 1, 6) and B(1, 3, 4).

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

88. (2)

Let image be (x_1, y_1, z_1)

$$\frac{x_1-1}{1} = \frac{y_1-3}{-1} = \frac{z_1-4}{1} = \frac{-2(1-3+4-5)}{3}$$

$x_1 = 3, y_1 = 1, z_1 = 6$

second statement is also true as (2, 2, 5) satisfies it.

89. Let $f : R \to R$ be a continuous function defined by

$$f(x) = \frac{1}{e^x + 2e^{-x}}$$

Statement–1 : $f(c) = \frac{1}{3}$, for some $c \in R$.

Statement–2 : $0 < f(x) \le \frac{1}{2\sqrt{2}}$, for all $x \in R$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

89. (1)

$$f(x) = \frac{1}{e^x + \frac{2}{e^x}} \qquad \left(\text{as } \frac{e^x + \frac{1}{e^x}}{2} \ge \sqrt{2} \right)$$

$\Rightarrow 0 < f(x) \le \frac{1}{2\sqrt{2}}$ and as $\frac{1}{3} < \frac{1}{2\sqrt{2}}$

$\Rightarrow f(c) = \frac{1}{3}$ as f(x) is continuous.

90. Let A be a 2 × 2 matrix with non–zero entries and let $A^2 = I$, where I is 2 × 2 identity matrix. Define

Tr(A) = sum of diagonal elements of A and |A| = determinant of matrix A.

Statement–1 : Tr(A) = 0

Statement–2 : |A| = 1.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(C) Statement-1 is True, Statement-2 is False.

(D) Statement-1 is False, Statement-2 is True.

90. (3)

$$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2+bc & ab+bd \\ ac+dc & bc+d^2 \end{bmatrix} = I$$

$\Rightarrow a^2 + bc = bc + d^2 = 1, (a + d)\, b = (a + d)c = 0$

$$\Rightarrow a = -d \Rightarrow A^2 = \begin{bmatrix} a^2+bc & 0 \\ 0 & a^2+bc \end{bmatrix}$$

$$= \left(a^2+bc\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$\Rightarrow a^2 + bc = 1$

$$|A| = \begin{vmatrix} a & b \\ c & -a \end{vmatrix} = -\left(a^2+bc\right) = -1 \text{ and } Tr(A) = a + d = 0$$

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(1)	31.	(4)	61.	(3)
2.	(2)	32.	(3)	62.	(2)
3.	(1)	33.	(1)	63.	(3)
4.	(3)	34.	(4)	64.	(4)
5.	(4)	35.	(2)	65.	(4)
6.	(1)	36.	(3)	66.	(2)
7.	(2)	37.	(3)	67.	(1)
8.	(3)	38.	(2)	68.	(2)
9.	(2)	39.	(2)	69.	(2)
10.	(3)	40.	(1)	70.	(4)
11	(4)	41.	(4)	71.	(1)
12.	(3)	42.	(3)	72.	(1)
13.	(2)	43.	(2)	73.	(1)
14.	(2)	44.	(3)	74.	(1)
15.	(2)	45.	(2)	75.	(3)
16.	(3)	46.	(4)	76.	(4)
17.	(3)	47.	(1)	77.	(3)
18.	(1)	48.	(4)	78.	(3)
19.	(3)	49.	(1)	79.	(2)
20.	(1)	50.	(1)	80.	(2)
21.	(4)	51.	(2)	81.	(2)
22.	(3)	52.	(3)	82.	(2)
23.	(1)	53.	(3)	83.	(3)
24.	(1)	54.	(3)	84.	(4)
25.	(4)	55.	(3)	85.	(4)
26.	(4)	56.	(2)	86.	(3)
27.	(4)	57.	(2)	87.	(3)
28.	(1)	58.	(4)	88.	(2)
29.	(4)	59.	(2)	89.	(1)
30.	(1)	60.	(3)	90.	(3)