FINAL JEE–MAIN EXAMINATION – FEBRUARY, 2021

(Held On Wednesday 24th February, 2021) TIME : 9 : 00 AM to 12 : 00 NOON

PHYSICS

SECTION-A

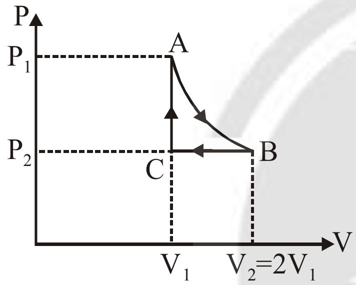
1. n mole a perfect gas undergoes a cyclic process ABCA (see figure) consisting of the following processes.

$A \rightarrow B$: Isothermal expansion at temperature T so that the volume is doubled from V_1 to $V_2 = 2V_1$ and pressure changes from P_1 to P_2.

$B \rightarrow C$: Isobaric compression at pressure P_2 to initial volume V_1.

$C \rightarrow A$: Isochoric change leading to change of pressure from P_2 to P_1.

Total workdone in the complete cycle ABCA is :



(1) 0 (2) $nRT\left(\ln 2+\frac{1}{2}\right)$

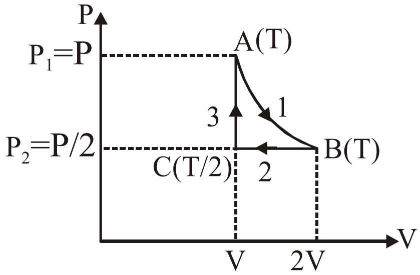
(3) nRTln2 (4) $nRT\left(\ln 2-\frac{1}{2}\right)$

Ans. (4)

Sol. $W_{Isothermal} = nRT\ln\left(\frac{v_2}{v_1}\right)$

$W_{Isobaric} = P\Delta V = nR\Delta T$

$W_{Isochoric} = 0$



$W_1 = nRT\ \ln\left(\frac{2V}{V}\right) = nRT\ \ln 2$

$W_2 = nR\left(\frac{T}{2}-T\right) = -nR\frac{T}{2}$

$W_3 = 0$

$\Rightarrow W_{net} = W_1 + W_2 + W_3$

$W_{net} = nRT\left(\ln 2-\frac{1}{2}\right)$

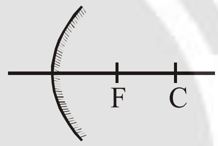
2. The focal length f is related to the radius of curvature r of the spherical convex mirror by:

(1) $f=+\frac{1}{2}r$ (2) $f = -r$

(3) $f = -\frac{1}{2}r$ (4) $f = r$

Ans. (1)

Sol. For convex mirror, focus is behind the mirror.



$\Rightarrow f = +\frac{r}{2}$

3. In a Young's double slit experiment, the width of the one of the slit is three times the other slit. The amplitude of the light coming from a slit is proportional to the slit-width. Find the ratio of the maximum to the minimum intensity in the interference pattern.

(1) 1 : 4 (2) 3 : 1 (3) 4 : 1 (4) 2 : 1

Ans. (3)

Sol. Amplitude $\propto$ Width of slit

$\Rightarrow A_2 = 3A_1$

$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{|\sqrt{I_1}-\sqrt{I_2}|}\right)^2$$

$\because$ Intensity $I \propto A^2$

$$\Rightarrow \frac{I_{max}}{I_{min}} = \left(\frac{A_1+A_2}{|A_1-A_2|}\right)^2$$

$$= \left(\frac{A_1+3A_1}{|A_1-3A_1|}\right)^2$$

$$= \left(\frac{4A_1}{2A_1}\right)^2 = 4 : 1$$

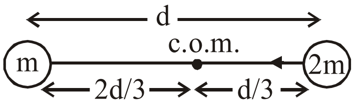
4. Two stars of masses m and 2m at a distance d rotate about their common centre of mass in free space. The period of revolution is :

(1) $\frac{1}{2\pi}\sqrt{\frac{d^3}{3Gm}}$ (2) $2\pi\sqrt{\frac{d^3}{3Gm}}$

(3) $\frac{1}{2\pi}\sqrt{\frac{3Gm}{d^3}}$ (4) $2\pi\sqrt{\frac{3Gm}{d^3}}$

Ans. (2)

Sol.



$$F = \frac{G(2m)m}{d^2} = (2m)\omega^2 (d/3)$$

$$\frac{Gm}{d^2} = \omega^2\frac{d}{3}$$

$$\Rightarrow \omega^2 = \frac{3Gm}{d^3}$$

$$\Rightarrow \omega = \sqrt{\frac{3Gm}{d^3}}$$

$$\Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{d^3}{3Gm}}$$

5. A current through a wire depends on time as $i = \alpha_0 t + \beta t^2$ where $\alpha_0 = 20$ A/s and $\beta = 8$ As^{-2}. Find the charge crossed through a section of the wire in 15 s.

(1) 2250 C (2) 11250 C

(3) 2100 C (4) 260 C

Ans. (2)

Sol. $i = 20t + 8t^2$

$$i = \frac{dq}{dt} \Rightarrow \int dq = \int i dt$$

$$\Rightarrow q = \int_0^{15}(20t + 8t^2)dt$$

$$q = \left(\frac{20t^2}{2} + \frac{8t^3}{3}\right)_0^{15}$$

$$q = 10 \times (15)^2 + \frac{8(15)^3}{3}$$

$q = 2250 + 9000$

$q = 11250$ C

6. Moment of inertia (M.I.) of four bodies, having same mass and radius, are reported as ;

I_1 = M.I. of thin circular ring about its diameter.

I_2 = M.I. of circular disc about an axis perpendicular to the disc and going through the centre,

I_3 = M.I. of solid cylinder about its axis and

I_4 = M.I. of solid sphere about its diameter.

Then :

(1) $I_1 + I_3 < I_2 + I_4$

(2) $I_1 + I_2 = I_3 + \frac{5}{2}I_4$

(3) $I_1 = I_2 = I_3 > I_4$

(4) $I_1 = I_2 = I_3 < I_4$

Ans. (3)

Sol. Ring $I_1 = \frac{MR^2}{2}$ about diameter

Disc $I_2 = \frac{MR^2}{2}$

Solid cylinder $I_3 = \frac{MR^2}{2}$

Solid sphere $I_4 = \frac{2}{5}MR^2$

$I_1 = I_2 = I_3 > I_4$

7. Given below are two statements :

Statement-I : Two photons having equal linear momenta have equal wavelengths.

Statement-II : If the wavelength of photon is decreased, then the momentum and energy of a photon will also decrease.

In the light of the above statements, choose the correct answer from the options given below.

(1) Both Statement I and Statement II are true

(2) Statement I is false but Statement II is true

(3) Both Statement I and Statement II are false

(4) Statement I is true but Statement II is false

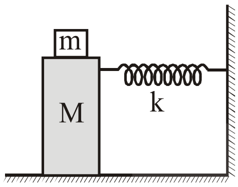
Ans. (4)

Sol. If linear momentum are equal then wavelength also equal

$$p = \frac{h}{\lambda},\ E = \frac{hc}{\lambda}$$

On decreasing wavelength, momentum and energy of photon increases.

8. In the given figure, a mass M is attached to a horizontal spring which is fixed on one side to a rigid support. The spring constant of the spring is k. The mass oscillates on a frictionless surface with time period T and amplitude A. When the mass is in equilibrium position, as shown in the figure, another mass m is gently fixed upon it. The new amplitude of oscillation will be :

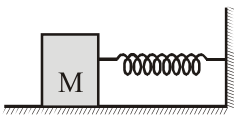


(1) $A\sqrt{\frac{M-m}{M}}$ (2) $A\sqrt{\frac{M}{M+m}}$

(3) $A\sqrt{\frac{M+m}{M}}$ (4) $A\sqrt{\frac{M}{M-m}}$

Ans. (2)

Sol.



Momentum of system remains conserved.

$p_i = p_f$

$MA\omega = (m + M)A'\omega'$

$$MA\sqrt{\frac{k}{M}} = (m+M)A'\sqrt{\frac{k}{m+M}}$$

$$A' = A\sqrt{\frac{M}{M+m}}$$

9. If Y, K and η are the values of Young's modulus, bulk modulus and modulus of rigidity of any material respectively. Choose the correct relation for these parameters.

(1) $Y = \frac{9K\eta}{3K-\eta} N/m^2$

(2) $\eta = \frac{3YK}{9K+Y} N/m^2$

(3) $Y = \frac{9K\eta}{2\eta+3K} N/m^2$

(4) $K = \frac{Y\eta}{9\eta-3Y} N/m^2$

Ans. (4)

Sol. Y- Younge modulus, K- Bulk modulus, η- modulus of rigidity

We know that

$y = 3k(1 - 2\sigma)$

$$\sigma = \frac{1}{2}\left(1-\frac{y}{3k}\right) \quad ....(i)$$

$y = 2\eta(1 + \sigma)$

$$\sigma = \frac{y}{2\eta}-1 \quad ....(ii)$$

From Eq.(i) and Eq. (ii)

$$\frac{1}{2}\left(1-\frac{Y}{3k}\right) = \frac{y}{2\eta} - 1$$

$$1 - \frac{y}{3k} = \frac{y}{\eta} - 2$$

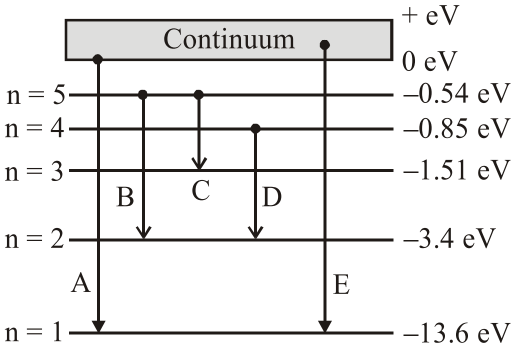
$$\frac{y}{3k} = 3 - \frac{y}{\eta}$$

$$\frac{y}{3k} = \frac{3\eta-y}{\eta}$$

$$\frac{\eta y}{3k} = 3\eta - y$$

$$k = \frac{\eta y}{9\eta-3y}$$

10. In the given figure, the energy levels of hydrogen atom have been shown along with some transitions marked A, B, C, D and E. The transitions A, B and C respectively represent :



(1) The ionization potential of hydrogen, second member of Balmer series and third member of Paschen series.

(2) The first member of the Lyman series, third member of Balmer series and second member of Paschen series.

(3) The series limit of Lyman series, third member of Balmer series and second member of Paschen series.

(4) The series limit of Lyman series, second member of Balmer series and second member of Paschen series.

Ans. (3)

Sol. A → Series limit of Lymen series.

B → Third member of Balmer series.

C → Second member of Paschen series.

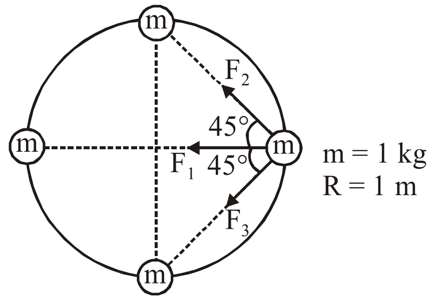
11. Four identical particles of equal masses 1kg made to move along the circumference of a circle of radius 1 m under the action of their own mutual gravitational attraction. The speed of each particle will be :

(1) $\sqrt{\frac{G}{2}(1+2\sqrt{2})}$ (2) $\sqrt{G(1+2\sqrt{2})}$

(3) $\sqrt{\frac{G}{2}(2\sqrt{2}-1)}$ (4) $\sqrt{\frac{(1+2\sqrt{2})G}{2}}$

Ans. (4)

Sol.



$$F_1 = \frac{Gmm}{(2R)^2} = \frac{Gm^2}{4R^2}$$

$$F_2 = \frac{Gmm}{(\sqrt{2}R)^2} = \frac{Gm^2}{2R^2}$$

$$F_3 = \frac{Gmm}{(\sqrt{2}R)^2} = \frac{Gm^2}{2R^2}$$

$$\Rightarrow F_{net} = F_1 + F_2 \cos 45° + F_3 \cos 45°$$

$$= \frac{Gm^2}{4R^2} + \frac{Gm^2}{2R^2}\frac{1}{\sqrt{2}} + \frac{Gm^2}{2R^2}\frac{1}{\sqrt{2}}$$

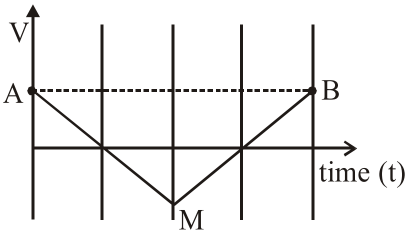
$$= \frac{Gm^2}{R^2}\left(\frac{1}{4} + \frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right)$$

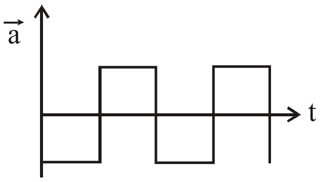
$$= \frac{Gm^2}{R^2}\left(\frac{1}{4} + \frac{1}{\sqrt{2}}\right) = \frac{Gm^2}{4R^2}\left(1+2\sqrt{2}\right)$$

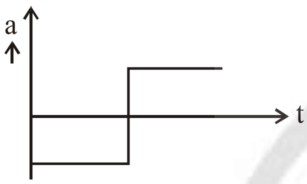
$$F_{net} = \frac{Gm^2}{4R^2}\left(1+2\sqrt{2}\right) = \frac{mv^2}{R}$$

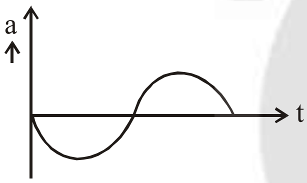
$$\Rightarrow v = \frac{\sqrt{G(1+2\sqrt{2})}}{2}$$

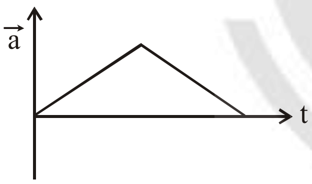
12. If the velocity-time graph has the shape AMB, what would be the shape of the corresponding acceleration-time graph ?



(1) 

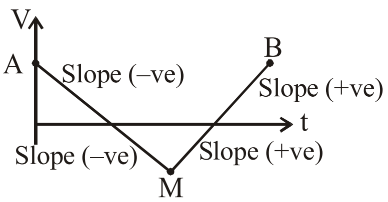
(2) 

(3) 

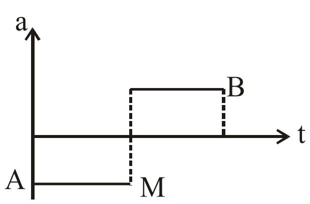
(4) 

Ans. (2)

Sol. Slope of v-t graph gives acceleration



$\Rightarrow$ Acceleration will be



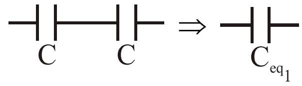
13. Two equal capacitors are first connected in series and then in parallel. The ratio of the equivalent capacities in the two cases will be:

(1) 4 : 1 (2) 2 : 1

(3) 1 : 4 (4) 1 : 2

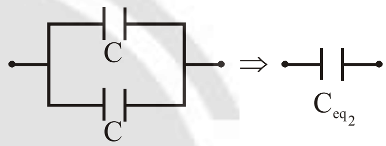
Ans. (3)

Sol. For series combination



$$\frac{1}{C_{eq_1}} = \frac{1}{C} + \frac{1}{C} \Rightarrow \boxed{C_{eq_1} = \frac{C}{2}}$$

For parallel combination



$$C_{eq_2} = C + C \Rightarrow C_{eq_2} = 2C$$

$$\Rightarrow \frac{C_{eq_1}}{C_{eq_2}} = \frac{(C/2)}{2C} = \frac{1}{4} = 1 : 4$$

14. If an emitter current is changed by 4 mA, the collector current changes by 3.5 mA. The value of β will be :

(1) 7 (2) 0.5

(3) 0.875 (4) 3.5

Ans. (1)

Sol. $I_\varepsilon = I_C + I_B$

$\Rightarrow \Delta I_\varepsilon = \Delta I_C + \Delta I_B$

$4mA = 3.5 \text{ mA} + \Delta I_B$

$\Rightarrow \Delta I_B = 0.5 \text{ mA}$

$$\Rightarrow \beta = \frac{\Delta I_C}{\Delta I_B}$$

$$\beta = \frac{3.5}{0.5}$$

$\Rightarrow \beta = 7$

15. Match List-I with List-II :

List-I	List-II
(a) Isothermal	(i) Pressure constant
(b) Isochoric	(ii) Temperature constant
(c) Adiabatic	(iii) Volume constant
(d) Isobaric	(iv) Heat content is constant

Choose the correct answer from the options given below :

(1) (a) $\rightarrow$ (i), (b) $\rightarrow$ (iii), (c) $\rightarrow$ (ii), (d) $\rightarrow$ (iv)
(2) (a) $\rightarrow$ (ii), (b) $\rightarrow$ (iii), (c) $\rightarrow$ (iv), (d) $\rightarrow$ (i)
(3) (a) $\rightarrow$ (ii), (b) $\rightarrow$ (iv), (c) $\rightarrow$ (iii), (d) $\rightarrow$ (i)
(4) (a) $\rightarrow$ (iii), (b) $\rightarrow$ (ii), (c) $\rightarrow$ (i), (d) $\rightarrow$ (iv)

Ans. (2)

Sol. (a) Isothermal $\Rightarrow$ Temperature constant
(a) $\rightarrow$ (ii)
(b) Isochoric $\Rightarrow$ Volume constant
(a) $\rightarrow$ (iii)
(c) Adiabatic $\Rightarrow \Delta Q = 0$
$\Rightarrow$ Heat content is constant
(c) $\rightarrow$ (iv)
(d) Isobaric $\Rightarrow$ Pressure constant
(d) $\rightarrow$ (i)

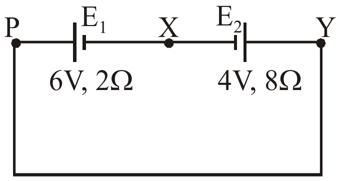
16. Each side of a box made of metal sheet in cubic shape is 'a' at room temperature 'T', the coefficient of linear expansion of the metal sheet is 'α'. The metal sheet is heated uniformly, by a small temperature ΔT, so that its new temperature is $T + \Delta T$. Calculate the increase in the volume of the metal box.

(1) $3a^3\alpha\Delta T$ (2) $4a^3\alpha\Delta T$
(3) $4\pi a^3\alpha\Delta T$ (4) $\frac{4}{3}\pi a^3\alpha\Delta T$

Ans. (1)

Sol. $\Delta V = V\gamma\Delta T$
$\Delta V = 3a^3\alpha\Delta T$

17. A cell E_1 of emf 6V and internal resistance 2Ω is connected with another cell E_2 of emf 4V and internal resistance 8Ω (as shown in the figure). The potential difference across points X and Y is :



(1) 10.0 V (2) 3.6 V
(3) 5.6V (4) 2.0 V

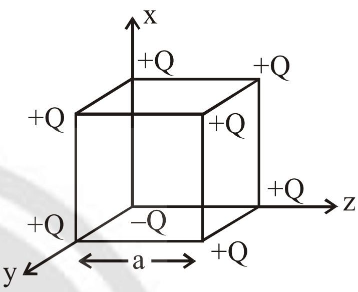
Ans. (3)

Sol. $I = \frac{6-4}{10} = \frac{1}{5}$ A

$V_x + 4 + 8 \times \frac{1}{5} - V_y = 0$

$V_x - V_y = -5.6 \Rightarrow |Vx - Vy| = 5.6$ V

18. A cube of side 'a' has point charges +Q located at each of its vertices except at the origin where the charge is –Q. The electric field at the centre of cube is :



(1) $\frac{-Q}{3\sqrt{3}\,\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$

(2) $\frac{-2Q}{3\sqrt{3}\,\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$

(3) $\frac{2Q}{3\sqrt{3}\,\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$

(4) $\frac{Q}{3\sqrt{3}\,\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$

Ans. (2)

Sol. We can replace –Q charge at origin by +Q and –2Q . Now due to +Q charge at every corner of cube. Electric field at center of cube is zero so now net electric field at center is only due to –2Q charge at origin.

$$\vec{E} = \frac{kq\vec{r}}{r^3} = \frac{1(-2Q)\frac{a}{2}(\hat{x}+\hat{y}+\hat{z})}{4\pi\varepsilon_0\left(\frac{a}{2}\sqrt{3}\right)^3}$$

$$\vec{E} = \frac{-2Q(\hat{x}+\hat{y}+\hat{z})}{3\sqrt{3}\pi a^2\varepsilon_0}$$

19. Consider two satellites S_1 and S_2 with periods of revolution 1 hr. and 8hr. respectively revolving around a planet in circular orbits. The ratio of angular velocity of satellite S_1 to the angular velocity of satellites S_2 is :

(1) 8 : 1 2) 1 : 4

(3) 2 : 1 (4) 1 : 8

Ans. (1)

Sol. $\frac{T_1}{T_2}=\frac{1}{8}$

$\frac{2\pi/\omega_1}{2\pi/\omega_2}=\frac{1}{8}$

$\frac{\omega_1}{\omega_2}=\frac{8}{1}$

20. The workdone by a gas molecule in an isolated system is given by, $W=\alpha\beta^2 e^{-\frac{x^2}{\alpha kT}}$, where x is the displacement, k is the Boltzmann constant and T is the temperature, α and β are constants. Then the dimension of β will be :

(1) $[M L^2 T^{-2}]$ (2) $[M L T^{-2}]$

(3) $[M^2 L T^2]$ (4) $[M^0 L T^0]$

Ans. (2)

Sol. $\frac{x^2}{\alpha kT}$ → dimensionless

$\Rightarrow [\alpha] = \frac{[x^2]}{[kT]} = \frac{L^2}{ML^2T^{-2}} = M^{-1}T^2$

Now $[W] = [\alpha]\,[\beta]^2$

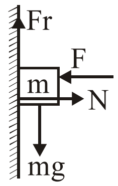
$[\beta] = \sqrt{\frac{ML^2T^{-2}}{M^{-1}T^2}} = M^1L^1T^{-2}$

SECTION-B

1. The coefficient of static friction between a wooden block of mass 0.5 kg and a vertical rough wall is 0.2. The magnitude of horizontal force that should be applied on the block to keep it adhere to the wall will be ______N. $[g = 10\ ms^{-2}]$

Ans. (25)

Sol. F.B.D. of the block is shown in the diagram



Since block is at rest therefore

fr – mg = 0 ....(1)

F – N = 0 ....(2)

fr ≤ μN

In limiting case

fr = μN = μF ....(3)

Using eq. (1) and (3)

∴ μF = mg

$\Rightarrow F = \frac{0.5\times10}{0.2} = 25$ N

Ans. 25.00

2. A resonance circuit having inductance and resistance 2×10^{-4} H and 6.28 Ω respectively oscillates at 10 MHz frequency. The value of quality factor of this resonator is _______. [π = 3.14]

Ans. (2000)

Sol. Given : $L = 2 \times 10^{-4}$ H

R = 6.28 Ω

f = 10 MHz = 10^7 Hz

Since quality factor,

$Q = \omega_0 \frac{L}{R} = 2\pi f \frac{L}{R}$

$\therefore Q = 2\pi \times 10^7 \times \frac{2\times10^{-4}}{6.28}$

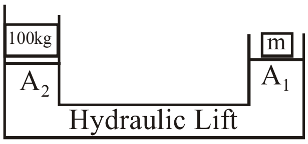
$Q = 2 \times 10^3 = 2000$

∴ Ans. is 2000

3. A hydraulic press can lift 100 kg when a mass 'm' is placed on the smaller piston. It can lift _______kg when the diameter of the larger piston is increased by 4 times and that of the smaller piston is decreased by 4 times keeping the same mass 'm' on the smaller piston.

Ans. (25600)

Sol. Using Pascals law



$$\frac{100 \times g}{A_2} = \frac{mg}{A_1} \quad ....(1)$$

Let m mass can lift M_0 in second case then

$$\frac{M_0 g}{16A_2} = \frac{mg}{A_1/16} \quad ....(2)$$

{Since A = $\frac{\pi d^2}{4}$ }

From equation (1) and (2) we get

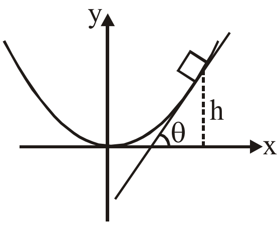
$$\frac{M_0}{16.100} = 16$$

$\Rightarrow M_0 = 25600$ kg

4. An inclined plane is bent in such a way that the vertical cross-section is given by $y = \frac{x^2}{4}$ where y is in vertical and x in horizontal direction. If the upper surface of this curved plane is rough with coefficient of friction $\mu = 0.5$, the maximum height in cm at which a stationary block will not slip downward is ________ cm.

Ans. (25)

Sol.



At maximum ht. block will experience maximum friction force. Therefore if at this height slope of the tangent is tan θ, then θ = Angle of repose.

$$\therefore \tan\theta = \frac{dy}{dx} = \frac{2x}{4} = \frac{x}{2} = 0.5$$

$\Rightarrow$ x = 1 and therefore $y = \frac{x^2}{4} = 0.25$ m

= 25 cm

$\therefore$ Answer is 25 cm

(Assuming that x & y in the equation are given in meter)

5. An electromagnetic wave of frequency 5 GHz, is travelling in a medium whose relative electric permittivity and relative magnetic permeability both are 2. Its velocity in this medium is ________ $\times 10^7$ m/s.

Ans. (15)

Sol. Given : Frequency of wave f = 5 GHz

= 5×10^9 Hz

Relative permittivity, $\in_r = 2$

and Relative permeability, $\mu_r = 2$

Since speed of light in a medium is given by,

$$v = \frac{1}{\sqrt{\mu \in}} = \frac{1}{\sqrt{\mu_r \mu_0 \cdot \in_r \in_0}}$$

$$v = \frac{1}{\sqrt{\mu_r \in_r}} \frac{1}{\sqrt{\mu_0 \in_0}} = \frac{C}{\sqrt{\mu_r \in_r}}$$

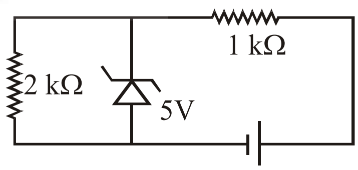
Where C is speed of light is vacuum.

$$\therefore v = \frac{3 \times 10^8}{\sqrt{4}} = \frac{30 \times 10^7}{2} \text{ m/s}$$

= 15×10^7 m/s

$\therefore$ Ans. is 15

6. In connection with the circuit drawn below, the value of current flowing through 2 kΩ resistor is ________ $\times 10^{-4}$ A.



Ans. (25)

Sol. Current through 2kΩ resistance

$$I = \frac{5}{2 \times 10^3} = 2.5 \times 10^{-3} \text{ A}$$

$I = 25 \times 10^{-4}$ A

Ans. 25

7. An audio signal $v_m = 20 \sin 2\pi (1500\ t)$ amplitude modulates a carrier

$v_C = 80 \sin 2\pi (100{,}000\ t)$.

The value of percent modulation is _______.

Ans. (25)

Sol. $\%\ \text{modulation} = \dfrac{Am}{Ac} \times 100$

$\%\ \text{modulation} = \dfrac{20}{80} \times 100$

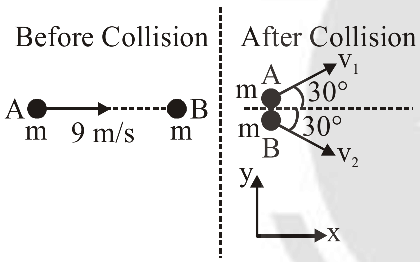
$\%\ \text{modulation} = 25\%$

Ans 25

8. A ball will a speed of 9 m/s collides with another identical ball at rest. After the collision, the direction of each ball makes an angle of 30° with the original direction. The ratio of velocities of the balls after collision is x : y, where x is ________.

Ans. (1)

Sol.



From conservation of momentum along y-axis.

$\vec{P}_{iy} = \vec{P}_{fy}$

$0 + 0 = mv_1 \sin 30°\,\hat{j} + mv_2 \sin 30°\,(-\hat{j})$

$mv_2 \sin 30° = mv_1 \sin 30°$

$v_2 = v_1$ or $\dfrac{v_1}{v_2} = 1$

Ans. 1

9. A common transistor radio set requires 12V (D.C.) for its operation. The D.C. source is constructed by using a transformer and a rectifier circuit, which are operated at 220 V (A.C.) on standard domestic A.C. supply. The number of turns of secondary coil are 24, then the nu mber of turns of primary are _______.

Ans. (440)

Sol. $\dfrac{N_P}{N_S} = \dfrac{V_P}{V_S}$

$\dfrac{N_P}{24} = \dfrac{220}{12}$

$N_P = \dfrac{220 \times 24}{12}$

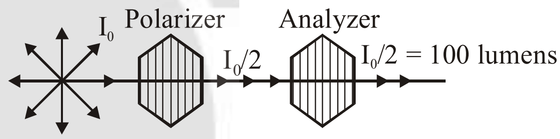
$N_P = 440$

Ans. 440 turns

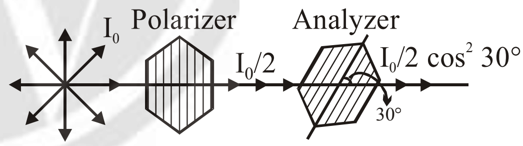
10. An unpolarized light beam is incident on the polarizer of a polarization experiment and the intensity of light beam emerging from the analyzer is measured as 100 Lumens. Now, if the analyzer is rotated around the horizontal axis (direction of light) by 30° in clockwise direction, the intensity of emerging light will be _______ Lumens.

Ans. (75)

Sol.



Assuming initially axis of Polarizer and Analyzer are parallel



Now emerging intensity $= \dfrac{I_0}{2} \cos^2 30°$

$= 100\left(\dfrac{\sqrt{3}}{2}\right)^2 = 100 \times \dfrac{3}{4} = 75$

Ans. 75

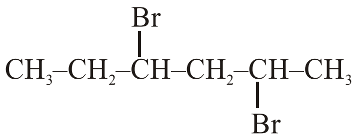
FINAL JEE–MAIN EXAMINATION – FEBRUARY, 2021

(Held On Wednesday 24th February, 2021) TIME : 9 : 00 AM to 12 : 00 NOON

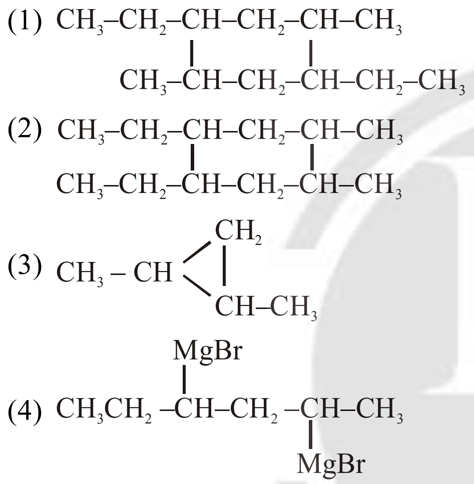
CHEMISTRY

SECTION-A

1. The product formed in the first step of the reaction of

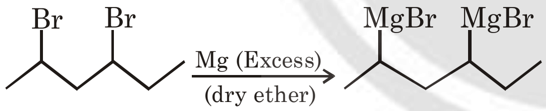


with excess $Mg/Et_2O(Et = C_2H_5)$ is :



Ans. (4)

Sol.



2. Consider the elements Mg. Al, S, P and Si, the correct increasing order of their first ionization enthalpy is :

(1) Mg < Al < Si < S < P
(2) Al < Mg < Si < S < P
(3) Mg < Al < Si < P < S
(4) Al < Mg < S < Si < P

Ans. (2)

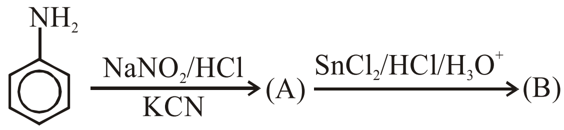
Sol. In general from left to right in a period, ionistion enthalpy increases due to effective nuclear charge increases.

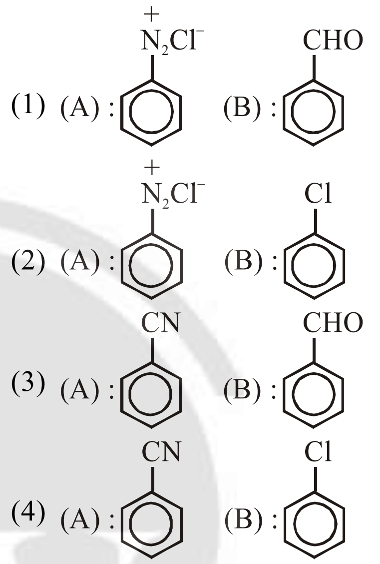
but due to extra stability of half filled and full filled electronic configuration, required ionisation enthalpy is more from neighbouring elements.

i.e. first ionisation enthalpy order is

Al < Mg < Si < S < P

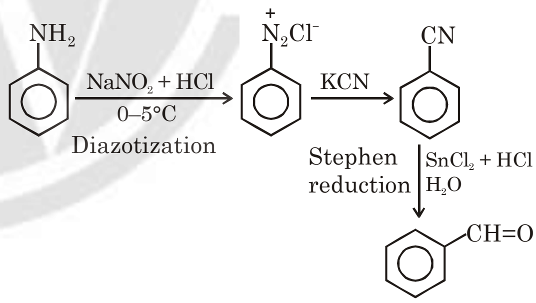
3. 'A' and 'B' in the following reactions are :





Ans. (3)

Sol.



4. Which of the following ore is concentrated using group 1 cyanide salt ?

(1) Sphalerite (2) Calamine
(3) Siderite (4) Malachite

Ans. (1)

Sol. Sphalerite ore : ZnS

Calamine ore : $ZnCO_3$

Siderite ore : $FeCO_3$

Malachite ore : $Cu(OH)_2.CuCO_3$

It is possible to separate two sulphide ores by adjusting proportion of oil to water or by using 'depressants'. In case of an ore containing ZnS and PbS, the depressant used is NaCN.

5. Al_2O_3 was leached with alkali to get X. The solution of X on passing of gas Y, forms Z. X, Y and Z respectively are :

(1) X = $Na[Al(OH)_4]$, Y = SO_2, Z = Al_2O_3
(2) X = $Na[Al(OH)_4]$, Y = CO_2, Z = $Al_2O_3.xH_2O$
(3) X = $Al(OH)_3$, Y = CO_2, Z = Al_2O_3
(4) X = $Al(OH)_3$, Y = SO_2, Z = $Al_2O_3.xH_2O$

Ans. (2)

Sol. $Al_2O_{3(s)} + 2NaOH_{(aq.)} + 3H_2O_{(\ell)}$

$\downarrow$

$2Na[Al(OH)_4]_{(aq.)}$ (X)

$\downarrow$ CO_2 (Y)

$Al_2O_3.xH_2O_{(s)} + 2NaHCO_{3(aq.)}$
(Z)

So

X : $Na[Al(OH)_4]$

Y : CO_2

Z : $Al_2O_3.xH_2O$

6. Which of the following are isostructural pairs ?

A. SO_4^{2-} and CrO_4^{2-}

B. $SiCl_4$ and $TiCl_4$

C. NH_3 and NO_3^-

D. BCl_3 and $BrCl_3$

BCl_3 and $BrCl_3$

(1) C and D only

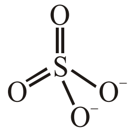
(2) A and B only

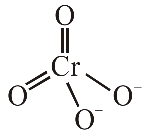
(3) A and C only

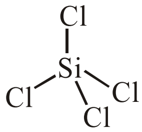
(4) B and C only

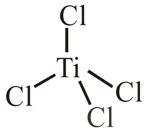
Ans. (2)

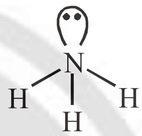
Sol. Isostructural means same structure

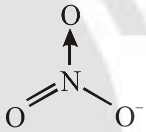
(A) SO_4^{2-}  : Tetrahedral

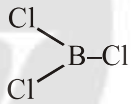
CrO_4^{2-}  : Tetrahedral

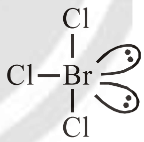
(B) $SiCl_4$  : Tetrahedral

$TiCl_4$  : Tetrahedral

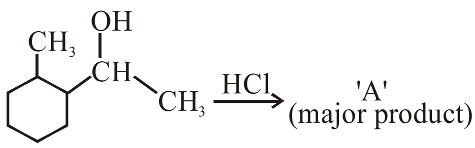
(C) NH_3  : Triagonal pyramidal

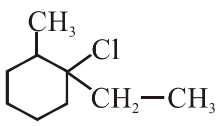
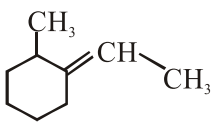
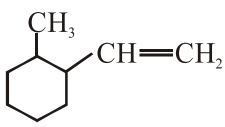
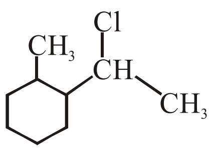
NO_3^-  : Triagonal planar

(D) BCl_3  : Triagonal planar

$BrCl_3$  : T-shape

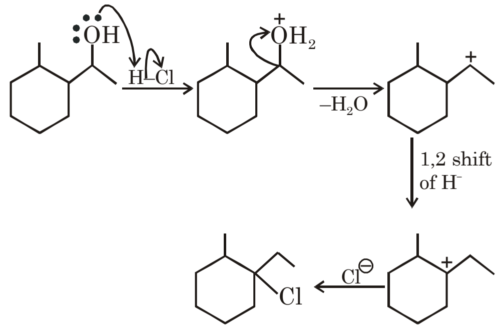
7. What is the final product (major) 'A' in the given reaction ?



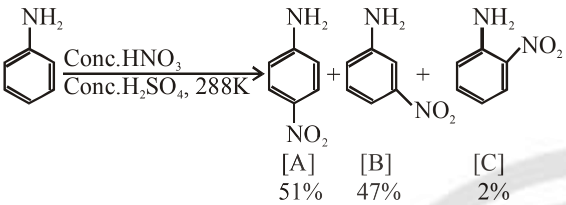
(1) 
(2) 
(3) 
(4) 

Ans. (1)

Sol.



8. In the following reaction the reason why meta-nitro product also formed is :



(1) low temperature

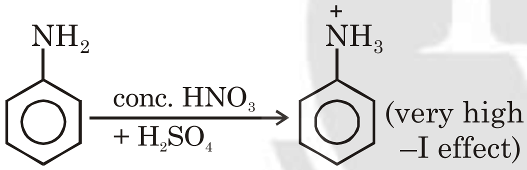
(2) $-NH_2$ group is highly meta-directive

(3) Formation of anilinium ion

(4) $-NO_2$ substitution always takes place at meta-position

Ans. (3)

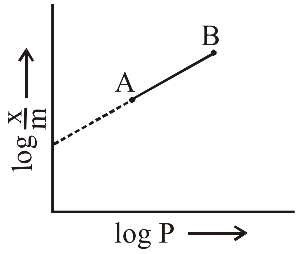
Sol.



(Anilinium ion)

Aniline on protonation gives anilinium ion which is meta directing. So considerable amount of meta product is formed.

9. In Freundlich adsorption isotherm, slope of AB line is :



(1) log n with (n > 1)

(2) n with (n, 0.1 to 0.5)

(3) $\log\frac{1}{n}$ with $(n<1)$

(4) $\frac{1}{n}$ with $\left(\frac{1}{n}=0 \text{ to } 1\right)$

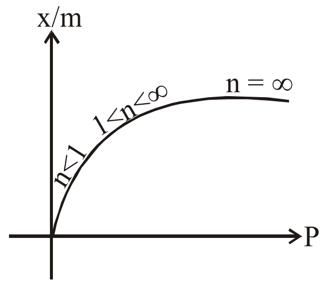
Ans. (4)

Sol. $\frac{x}{m}=K(P)^{1/n}$

$$\log\left(\frac{x}{m}\right)=\log K+\frac{1}{n}\log P$$

y = c + mx

m = 1/n so slope will be equal to 1/n.



Hence $\boxed{0\le\frac{1}{n}\le 1}$

10. (A) $HOCl + H_2O_2 \rightarrow H_3O^+ + Cl^- + O_2$

(B) $I_2 + H_2O_2 + 2OH^- \rightarrow 2I^- + 2H_2O + O_2$

Choose the correct option.

(1) H_2O_2 acts as reducing and oxidising agent respectively in equation (A) and (B)

(2) H_2O_2 acts as oxidising agent in equation (A) and (B)

(3) H_2O_2 acts as reducing agent in equation (A) and (B)

(4) H_2O_2 act as oxidizing and reducing agent respectively in equation (A) and (B)

Ans. (3)

(A) $HOCl+H_2O_2 \rightarrow H_3O^+ + Cl^- + O_2$

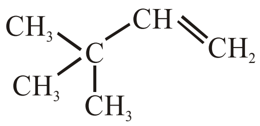
In this equation, H_2O_2 is reducing chlorine from +1 to –1.

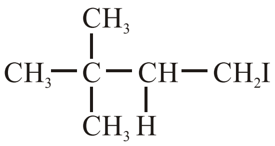
(B) $I_2 + H_2O_2 + 2OH^- \rightarrow 2I^- + 2H_2O + O_2$

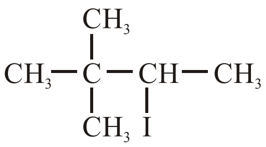
In this equation, H_2O_2 is reducing iodine from 0 to –1.

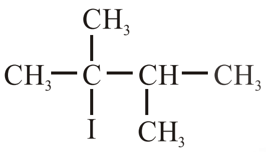
Sol. In (A) reduction of HOCl occurs so it will be a oxidising agent hence H_2O_2 will be a reducing agent.

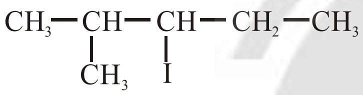
In(B) reduction of I_2 occurs so it will be a oxidising agent and H_2O_2 will be a reducing agent.

11. What is the major product formed by HI on reaction with  ?

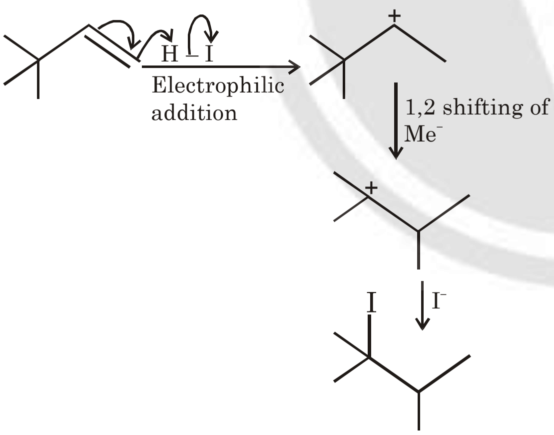
(1) 

(2) 

(3) 

(4) 

Ans (3)

Sol. 

12. Which of the following reagent is used for the following reaction ?

$$CH_3CH_2CH_3 \xrightarrow{?} CH_3CH_2CHO$$

(1) Manganese acetate
(2) Copper at high temperature and pressure
(3) Molybdenum oxide
(4) Potassium permanganate

Ans (3)

Sol. $CH_3-CH_2-CH_3 \xrightarrow{MO_2O_3} CH_3-CH_2-CH{=}O$

The reagent used will be MO_2O_3

13. Given below are two statements :

Statement I : Colourless cupric metaborate is reduced to cuprous metaborate in a luminous flame.

Statement II : Cuprous metaborate is obtained by heating boric anhydride and copper sulphate in a non-luminous flame.

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) Statement I is true but Statement II is false
(2) Both Statement I and Statement II are false
(3) Statement I is false but Statement II is true
(4) Both Statement I and Statement II are true

Ans. (2)

Sol.

(i) Blue cupric metaborate is reduced to colourless cuprous metaborate in a luminous flame

$$2Cu(BO_2)_2 + 2NaBO_2 + C \xrightarrow{\text{Luminous flame}} 2CuBO_2 + Na_2B_4O_7 + CO$$

(ii) Cupric metaborate is obtained by heating boric anhydride and copper sulphate in a non luminous flame.

$$CuSO_4 + B_2O_3 \xrightarrow[\text{Flame}]{\text{Non-luminous}} \underset{\text{Cupric metaborate (Blue-green)}}{Cu(BO_2)_2 + SO_3}$$

14. Out of the following, which type of interaction is responsible for the stabilisation of α-helix structure of proteins ?

(1) Ionic bonding

(2) Hydrogen bonding

(3) Covalent bonding

(4) vander Waals forces

Ans. (2)

Sol. Hydrogen bonding is responsible for the stacking of α-helix structure of protein.

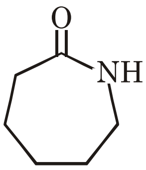
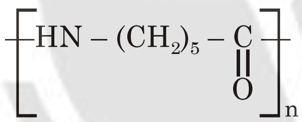
15. Match List I with List II.

List I (Monomer Unit)	List II (Polymer)
(a) Caprolactum	(i) Natural rubber
(b) 2-Chloro-1,3-butadiene	(ii) Buna-N
(c) Isoperene	(iii) Nylon 6
(d) Acrylonitrile	(iv) Neoprene

Choose the correct answer from the options given below :

(1) (a) → (iv), (b) → (iii), (c) → (ii), (d) → (i)
(2) (a) → (ii), (b) → (i), (c) → (iv), (d) → (iii)
(3) (a) → (iii), (b) → (iv), (c) → (i), (d) → (ii)
(4) (a) → (i), (b) → (ii), (c) → (iii), (d) → (iv)

Ans. (3)

Sol. (a)  Caprolactum is the monomeric unit of polymer Nylon-6 

(b) 2-Chlorobuta-1, 3-diene is the monomeric unit of polymer neoprene.

(c) 2-Methylbuta-1, 3-diene is the monomeric unit of polymer natural rubber.

(d) $CH_2 = CH - CN$ (Acrylonitrile) is the one of the monomeric unit of polymer Buna-N

16. The gas released during anaerobic degradation of vegetation may lead to :

(1) Ozone hole
(2) Acid rain
(3) Corrosion of metals
(4) Global warming and cancer

Ans. (4)

Sol. The gas CH_4 evolved due to anaerobic degradation of vegetation which causes global warming and cancer.

17. The major components in "Gun Metal" are :

(1) Cu, Zn and Ni (2) Cu, Sn and Zn
(3) Al, Cu, Mg and Mn (4) Cu, Ni and Fe

Ans. (2)

The major components in "Gun Metal" are
Cu : 87%
Zn : 3%
Sn : 10%

18. The electrode potential of M^{2+} / M of 3d-series elements shows positive value of :

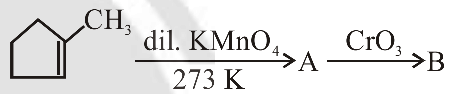
(1) Zn (2) Fe (3) Co (4) Cu

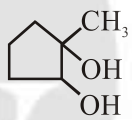
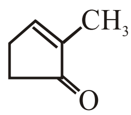
Ans.(4)

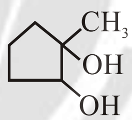
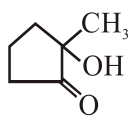
Sol. Only copper shows positive value for electrode potential of M^{2+}/M of 3d-series elements.

$E^{\ominus} / V_{(Cu^{2+}/Cu)}$: +0.34

19. Identify products A and B :

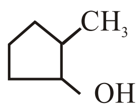
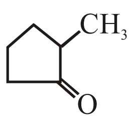


(1) A :  B : 

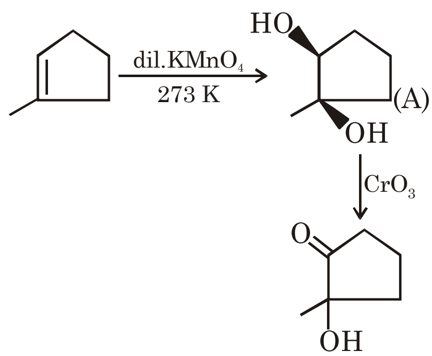
(2) A :  B : 

(3) A : $OHC—CH_2CH_2CH_2—\overset{O}{\overset{\|}{C}}—CH_3$

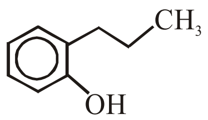
B : $HOOC—CH_2CH_2CH_2—\overset{O}{\overset{\|}{C}}—CH_3$

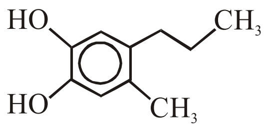
(4) A :  (4) 

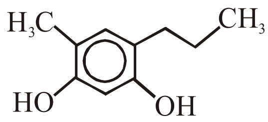
Ans. (2)

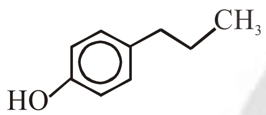
Sol. 

20. Which of the following compound gives pink colour on reaction with phthalic anhydride in conc. H_2SO_4 followed by treatment with NaOH ?

(1) 

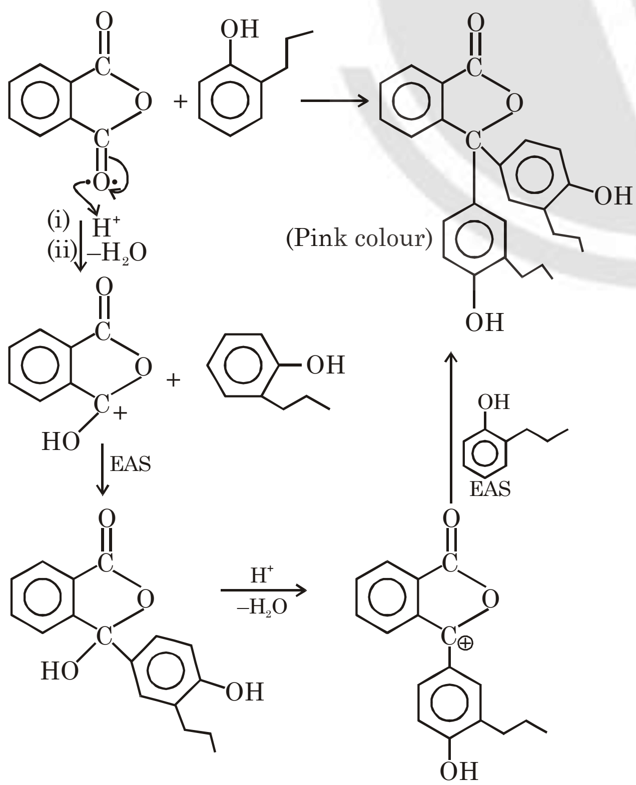
(2) 

(3) 

(4) 

Ans. (1)

Sol.



SECTION-B

1. When 9.45 g of $ClCH_2COOH$ is added to 500 mL of water, its freezing point drops by 0.5°C. The dissociation constant of $ClCH_2COOH$ is $x \times 10^{-3}$. The value of x is ________. (Rounded off to the nearest integer)

$\left[K_{f(H_2O)} = 1.86\,K\,kg\,mol^{-1}\right]$

Ans. (36)

Sol. $ClCH_2COOH \rightleftharpoons ClCH_2COO^{\ominus} + H^+$

$i = 1 + (2 - 1)\,\alpha$

$i = 1 + \alpha$

$\Delta T_f = ik_f m$

$$0.5 = (1+\alpha)(1.86)\left(\frac{\left(\frac{9.45}{94.5}\right)}{\left(\frac{500}{1000}\right)}\right)$$

$$\frac{5}{3.72} = 1 + \alpha \quad \Rightarrow \alpha = \frac{1.28}{3.72}$$

$$\alpha = \frac{32}{93}$$

$ClCH_2COOH \rightleftharpoons ClCH_2\,COO^{\ominus} + H^+$

$C - C\alpha \qquad C\alpha \qquad C\alpha$

$$K_a = \frac{(C\alpha)^2}{C - C\alpha} = \frac{C\alpha^2}{1-\alpha} \qquad C = \frac{0.1}{500/1000} = 0.2$$

$$K_a = \frac{0.2(32/93)^2}{(1-32/93)} = \frac{0.2 \times (32)^2}{93 \times 61}$$

$= 0.036$

$K_a = 36 \times 10^{-3}$

2. 4.5 g of compound A (MW = 90) was used to make 250 mL of its aqueous solution. The molarity of the solution in M is $x \times 10^{-1}$. The value of x is ________. (Rounded off to the nearest integer)

Ans. (2)

Sol. $M = \dfrac{4.5/90}{250/1000} = 0.2$

$= 2 \times 10^{-1}$

3. At 1990 K and 1 atm pressure, there are equal number of Cl_2 molecules and Cl atoms in the reaction mixture. The value K_P for the reaction $Cl_{2(g)} \rightleftharpoons 2Cl_{(g)}$ under the above conditions is $x \times 10^{-1}$. The value of x is _______. (Rounded of to the nearest integer)

Ans. (5)

Sol. $Cl_2 \rightleftharpoons 2Cl$

Let mol of both of Cl_2 and Cl is x

$$P_{Cl} = \frac{x}{2x} \times 1 = \frac{1}{2}$$

$$P_{Cl_2} = \frac{x}{2x} \times 1 = \frac{1}{2}$$

$$K_p = \frac{\left(\frac{1}{2}\right)^2}{\frac{1}{2}} = \frac{1}{2} = 0.5 \Rightarrow 5 \times 10^{-1}$$

4. Number of amphoteric compound among the following is __________

(A) BeO (B) BaO

(C) $Be(OH)_2$ (D) $Sr(OH)_2$

Ans. (2)

Sol. Both compounds BeO and $Be(OH)_2$ are amphoteric in nature.

and both compounds BaO and $Sr(OH)_2$ are basic in nature.

5. The reaction of sulphur in alkaline medium is the below:

$$S_{8(s)} + a\ OH^-_{(aq)} \rightarrow b\ S^{2-}_{(aq)} + c\ S_2O_3^{2-}{}_{(aq)} + d\ H_2O_{(\ell)}$$

The values of 'a' is _______. (Integer answer)

Ans. (12)

Sol.

$$16e^{\ominus} + S_8 \longrightarrow 8S^{2-}$$

$$12H_2O + S_8 \longrightarrow 4S_2O_3^{2-} + 24H^+ + 16e^{\ominus}$$

$$2S_8 + 12\ H_2O \longrightarrow 8S^{2-} + 4S_2O_3^{2-} + 24\ H^+$$

for balancing in basic medium add equal number of $OH^{\ominus}$ that of H^+

$2S_8 + 12H_2O + 24OH^{\ominus} \longrightarrow 8S^{2-} + 4\ S_2O_8^{2-} + 24H_2O$

$2S_8 + 24OH^{\ominus} \rightarrow 8S^{2-} + 4S_2O_8^{2-} + 12\ H_2O$

$S_8 + 12\ OH^{\ominus} \rightarrow 4S^{2-} + 2S_2O_8^{2-} + 6\ H_2O$

a = 12

6. For the reaction $A_{(g)} \rightarrow (B)_{(g)}$, the value of the equilibrium constant at 300 K and 1 atm is equal to 100.0. The value of Δ_rG for the reaction at 300 K and 1 atm in J mol^{-1} is – xR, where x is ______ (Rounded of to the nearest integer) (R = 8.31 J mol^{-1} K^{-1} and ln 10 = 2.3)

Ans. (1380)

6. $\Delta G^o = -RT \ln Kp$

$= -R(300)\ (2)\ \ln(10)$

$= -R(300 \times 2 \times 2.3)$

$\Delta G^o = -1380\ R$

7. A proton and a Li^{3+} nucleus are accelerated by the same potential. If λ_{Li} and λ_P denote the de Broglie wavelengths of Li^{3+} and proton respectively, then the value of $\frac{\lambda_{Li}}{\lambda_P}$ is $x \times 10^{-1}$. The value of x is ________.

(Rounded off to the nearest integer) (Mass of Li^{3+} = 8.3 mass of proton)

Ans. (2)

Sol. $\lambda = \dfrac{h}{\sqrt{2mqV}}$

$$\frac{\lambda_{Li}}{\lambda_p} = \sqrt{\frac{m_p(e)V}{m_{Li}(3e)(V)}} \qquad m_{Li} = 8.3\,m_p$$

$$\frac{\lambda_{Li}}{\lambda_p} = \sqrt{\frac{1}{8.3\times 3}} = \frac{1}{5} = 0.2 = 2\times10^{-1}$$

8. The stepwise formation of $[Cu(NH_3)_4]^{2+}$ is given below

$$Cu^{2+} + NH_3 \xrightleftharpoons{K_1} [Cu(NH_3)]^{2+}$$

$$[Cu(NH_3)]^{2+} + NH_3 \xrightleftharpoons{K_2} [Cu(NH_3)_2]^{2+}$$

$$[Cu(NH_3)_2]^{2+} + NH_3 \xrightleftharpoons{K_3} [Cu(NH_3)_3]^{2+}$$

$$[Cu(NH_3)_3]^{2+} + NH_3 \xrightleftharpoons{K_4} [Cu(NH_3)_4]^{2+}$$

The value of stability constants K_1, K_2, K_3 and K_4 are 10^4, 1.58×10^3, 5×10^2 and 10^2 respectively. The overall equilibrium constants for dissociation of $[Cu(NH_3)_4]^{2+}$ is $x \times 10^{-12}$. The value of x is ________. (Rounded off to the nearest integer)

Ans. (1)

Sol. $Cu^{2+} + NH_3 \xrightleftharpoons{K_1} [Cu(NH_3)]^{2+}$

$$[Cu(NH_3)]^{2+} + NH_3 \xrightleftharpoons{K_2} [Cu(NH_3)_2]^{2+}$$

$$[Cu(NH_3)_2]^{2+} + NH_3 \xrightleftharpoons{K_3} [Cu(NH_3)_3]^{2+}$$

$$[Cu(NH_3)_3]^{2+} + NH_3 \xrightleftharpoons{K_4} [Cu(NH_3)_4]^{2+}$$

$$Cu^{2+} + 4NH_3 \xrightleftharpoons{K} [Cu(NH_3)_4]^{2+}$$

So

$K = K_1 \times K_2 \times K_3 \times K_4$

$= 10^4 \times 1.58 \times 10^3 \times 5 \times 10^2 \times 10^2$

$K = 7.9 \times 10^{11}$

Where K → Equilibrium constant for formation of $[Cu(NH_3)_4]^{2+}$

So equilibrium constant (K') for dissociation of $[Cu(NH_3)_4]^{2+}$ is $\dfrac{1}{K}$

$$K' = \frac{1}{K}$$

$$K' = \frac{1}{7.9\times10^{11}}$$

$= 1.26 \times 10^{-12} = (x \times 10^{-12})$

So the value of x = 1.26

OMR Ans = 1 (After rounded off to the nearest integer)

9. The coordination number of an atom in a body-centered cubic structure is _______.

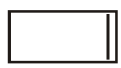
[Assume that the lattice is made up of atoms.]

Ans. (8)

Sol. 8

10. Gaseous cyclobutene isomerizes to butadiene in a first order process which has a 'k' value of $3.3 \times 10^{-4} s^{-1}$ at 153°C. The time in minutes it takes for the isomerization to proceed 40 % to completion at this temperature is _____. (Rounded off to the nearest integer)

Ans. (26)

Sol.  $\longrightarrow H_2C = HC–CH = CH_2$

$$Kt = \ell n\frac{[A]_0}{[A]_t}$$

$$3.3\times10^{-4}\times t = \ell n\left(\frac{100}{60}\right)$$

t = 1547.956 sec

t = 25.799 min

26 min

FINAL JEE–MAIN EXAMINATION – FEBRUARY, 2021

(Held On Wednesday 24th February, 2021) TIME : 9 : 00 AM to 12 : 00 NOON

MATHEMATICS

SECTION-A

1. The statement among the following that is a tautology is :

(1) $A \vee (A \wedge B)$

(2) $A \wedge (A \vee B)$

(3) $B \rightarrow [A \wedge (A \rightarrow B)]$

(4) $[A \wedge (A \rightarrow B)] \rightarrow B$

Ans. (4)

Sol. $\left(A \wedge (A \rightarrow B)\right) \rightarrow B$

$= \left(A \wedge (\sim A \vee B)\right) \rightarrow B$

$= \left((A \wedge \sim A) \vee (A \wedge B)\right) \rightarrow B$

$= (A \wedge B) \rightarrow B$

$= \sim (A \wedge B) \vee B$

$= (\sim A \vee \sim B) \vee B$

$= T$

2. A man is walking on a straight line. The arithmetic mean of the reciprocals of the intercepts of this line on the coordinate axes is $\frac{1}{4}$. Three stones A, B and C are placed at the points (1,1), (2, 2) and (4, 4) respectively. Then which of these stones is / are on the path of the man ?

(1) A only (2) C only

(3) All the three (4) B only

Ans (4)

Sol. Let the line be y = mx + c

x-intercept : $-\frac{c}{m}$

y-intercept : c

A.M of reciprocals of the intercepts :

$$\frac{-\frac{m}{c} + \frac{1}{c}}{2} = \frac{1}{4} \Rightarrow 2(1-m) = c$$

line : y = mx + 2(1 – m) = c

$\Rightarrow$ (y – 2) – m(x – 2) = 0

$\Rightarrow$ line always passes through (2, 2)

Ans. 4

3. The equation of the plane passing through the point (1, 2, –3) and perpendicular to the planes 3x + y – 2z = 5 and 2x – 5y – z = 7, is

(1) 3x – 10y – 2z + 11 = 0

(2) 6x – 5y – 2z – 2 = 0

(3) 11x + y + 17z + 38 = 0

(4) 6x – 5y + 2z + 10 = 0

Ans. (3)

Sol. Normal vector :

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 2 & -5 & -1 \end{vmatrix} = -11\hat{i} - \hat{j} + 17\hat{k}$$

So drs of normal to the required plane is <11, 1, 17>

plane passes through (1, 2, –3)

So eq^n of plane :

11(x – 1) + 1(y – 2) + 17(z + 3) = 0

$\Rightarrow$ 11x + y + 17z + 38 = 0

4. The population P = P(t) at time 't' of a certain species follows the differential equation $\frac{dP}{dt} = 0.5P - 450$. If P(0) = 850, then the time at which population becomes zero is :

(1) $\log_e 18$ (2) $\log_e 9$

(3) $\frac{1}{2}\log_e 18$ (4) $2\log_e 18$

Ans. (4)

Sol. $\frac{dP}{dt} = 0.5P - 450$

$$\Rightarrow \int_0^t \frac{dp}{P-900} = \int_0^t \frac{dt}{2}$$

$$\Rightarrow \left[\ell n|P(t)-900|\right]_0^t = \left[\frac{t}{2}\right]_0^t$$

$\Rightarrow \quad \ell n|P(t)-900|-\ell n|P(0)-900|=\frac{t}{2}$

$\Rightarrow \quad \ell n|P(t)-900|-\ell n|50|=\frac{t}{2}$

for P(t) = 0

$\Rightarrow \quad \ell n\left|\frac{900}{50}\right|=\frac{t}{2} \Rightarrow t=2\ell n18$

5. The system of linear equations
$3x - 2y - kz = 10$
$2x - 4y - 2z = 6$
$x + 2y - z = 5m$
is inconsistent if :

(1) $k=3, m=\frac{4}{5}$ (2) $k \neq 3, m \in R$

(3) $k \neq 3, m \neq \frac{4}{}$ (4) $k=3, m \neq \frac{4}{5}$

Ans. (4)

Sol. $\Delta=\begin{vmatrix}3 & -2 & -k\\ 2 & -4 & -2\\ 1 & 2 & -1\end{vmatrix}=0$

$\Rightarrow$ 24 – 2(0) – k(8) = 0 $\Rightarrow$ k = 3

$\Delta_x=\begin{vmatrix}10 & -2 & -3\\ 6 & -4 & -2\\ 5m & 2 & -1\end{vmatrix}$

= 10(8) – 2(–10m + 6) – 3(12 + 20m)
= 8(4 – 5m)

$\Delta_y=\begin{vmatrix}3 & 10 & -3\\ 2 & 6 & -2\\ 1 & 5m & -1\end{vmatrix}$

= 3(–6 + 10m) + 10(0) – 3(10m – 6)
= 0

$\Delta_z=\begin{vmatrix}3 & -2 & 10\\ 2 & -4 & 6\\ 1 & 2 & 5m\end{vmatrix}$

= 3(–20m – 12) – 2(6 – 10m) + 10(8)
= 40m – 32 = 8(5m – 4)
for inconsistent

k = 3 & $m \neq \frac{4}{5}$

6. If f : R $\to$ R is a function defined by $f(x)=[x-1]\cos\left(\frac{2x-1}{2}\right)\pi,$ where [.] denotes the greatest integer function, then f is :

(1) discontinuous at all integral values of x except at x = 1

(2) continuous only at x = 1

(3) continuous for every real x

(4) discontinuous only at x = 1

Ans. (3)

Sol. For x = n, n $\in$ Z

$LHL=\lim_{x\to n^-} f(x)=\lim_{x\to n^-}[x-1]\cos\left(\frac{2x-1}{2}\right)\pi$

= 0

$RHL=\lim_{x\to n^+} f(x)=\lim_{x\to n^+}[x-1]\cos\left(\frac{2x-1}{2}\right)\pi$

= 0

$f(n)$ = 0

$\Rightarrow$ LHL = RHL = $f(n)$

$\Rightarrow$ $f(x)$ is continuous for every real x.

7. The distance of the point (1, 1, 9) from the point of intersection of the line $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}$ and the plane x + y + z = 17 is :

(1) $2\sqrt{19}$ (2) $19\sqrt{2}$

(3) 38 (4) $\sqrt{38}$

Ans. (4)

Sol. Let $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}=t$

$\Rightarrow$ x = 3 + t, y = 2t + 4, z = 2t + 5
for point of intersection with x + y + z = 17
3 + t + 2t + 4 + 2t + 5 = 17
$\Rightarrow$ 5t = 5 $\Rightarrow$ t = 1
$\Rightarrow$ point of intersection is (4, 6, 7)
distance between (1, 1, 9) and (4, 6, 7)
is $\sqrt{9+25+4}=\sqrt{38}$

8. If the tangent to the curve $y = x^3$ at the point $P(t, t^3)$ meets the curve again at Q, then the ordinate of the point which divides PQ internally in the ratio 1 : 2 is :

(1) $-2t^3$ (2) 0 (3) $-t^3$ (4) $2t^3$

Ans. (1)

Sol. Slope of tangent at $P(t, t^3)$ = $\left.\frac{dy}{dx}\right]_{(t,t^3)}$

$= (3x^2)_{x=t} = 3t^2$

So equation tangent at $P(t, t^3)$:

$y - t^3 = 3t^2(x - t)$

for point of intersection with $y = x^3$

$x^3 - t^3 = 3t^2x - 3t^3$

$\Rightarrow \quad (x - t)(x^2 + xt + t^2) = 3t^2(x - t)$

for $x \neq t$

$x^2 + xt + t^2 = 3t^2$

$\Rightarrow \quad x^2 + xt - 2t^2 = 0 \quad \Rightarrow \quad (x - t)(x + 2t) = 0$

So for Q : $x = -2t$, $Q(-2t, -8t^3)$

ordinate of required point : $\frac{2t^3 - 8t^3}{2+1} = -2t^3$

9. If $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx = a \sin^{-1}\left(\frac{\sin x + \cos x}{b}\right) + c$, where c is a constant of integration, then the ordered pair (a, b) is equal to :

(1) (–1, 3) (2) (3, 1)

(3) (1, 3) (4) (1, –3)

Ans. (3)

Sol. $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx$

$= \int \frac{\cos x - \sin x}{\sqrt{9 - (\sin x + \cos x)^2}} dx$

Let $\sin x + \cos x = t$

$\int \frac{dt}{\sqrt{9 - t^2}} = \sin^{-1}\frac{t}{3} + c$

$= \sin^{-1}\left(\frac{\sin x + \cos x}{3}\right) + c$

So a = 1, b = 3.

10. The value of $-{}^{15}C_1 + 2.{}^{15}C_2 - 3.{}^{15}C_3 + \ldots\ldots -15.{}^{15}C_{15} + {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + \ldots. + {}^{14}C_{11}$ is :

(1) $2^{16} - 1$ (2) $2^{13} - 14$

(3) 2^{14} (4) $2^{13} - 13$

Ans. (2)

Sol. $(-{}^{15}C_1 + 2.{}^{15}C_2 - 3.{}^{15}C_3 + \ldots\ldots -15.{}^{15}C_{15}) + ({}^{14}C_1 + {}^{14}C_3 + \ldots. + {}^{14}C_{11})$

$= \sum_{r=1}^{15} (-1)^r.r\,{}^{15}C_r + \left({}^{14}C_1 + {}^{14}C_3 + \ldots + {}^{14}C_{11} + {}^{14}C_{13}\right) - {}^{14}C_3$

$= \sum_{r=1}^{15} (-1)^r\, 15.\,{}^{14}C_{r-1} + 2^{13} - 14$

$= 15(-{}^{14}C_0 + {}^{14}C_1 \ldots\ldots - {}^{14}C_{14}) + 2^{13} - 14$

$= 2^{13} - 14$

11. The function

$f(x) = \frac{4x^3 - 3x^2}{6} - 2\sin x + (2x - 1)\cos x$:

(1) increases in $\left[\frac{1}{2}, \infty\right)$

(2) increases in $\left(-\infty, \frac{1}{2}\right]$

(3) decreases in $\left[\frac{1}{2}, \infty\right)$

(4) decreases in $\left(-\infty, \frac{1}{2}\right]$

Ans. (1)

Sol. $f(x) = \frac{4x^3 - 3x^2}{6} - 2\sin x + (2x - 1)\cos x$

$f'(x) = (2x^2 - x) - 2\cos x + 2\cos x - \sin x(2x - 1)$

$= (2x - 1)(x - \sin x)$

for $x > 0$, $x - \sin x > 0$

$x < 0$, $x - \sin x < 0$

for $x \in (-\infty, 0] \cup \left[\frac{1}{2}, \infty\right)$, $f'(x) \geq 0$

for $x \in \left[0, \frac{1}{2}\right]$, $f'(x) \leq 0$

$\Rightarrow \quad f(x)$ increases in $\left[\frac{1}{2}, \infty\right)$.

12. Let $f : R \to R$ be defined as $f(x) = 2x - 1$ and $g : R - \{1\} \to R$ be defined as $g(x) = \dfrac{x - \frac{1}{2}}{x-1}$.

Then the composition function $f(g(x))$ is :

(1) onto but not one-one

(2) both one-one and onto

(3) one-one but not onto

(4) neither one-one nor onto

Ans. (3)

Sol. $f(g(x)) = 2g(x) - 1 = 2\left(\dfrac{2x-1}{2(x-1)}\right) - 1$

$$= \frac{x}{x-1} = 1 + \frac{1}{x-1}$$

Range of $f(g(x) = \mathbb{R} - \{1\}$

Range of $f(g(x))$ is not onto

& $f(g(x))$ is one-one

So $f(g(x))$ is one-one but not onto.

13. An ordinary dice is rolled for a certain number of times. If the probability of getting an odd number 2 times is equal to the probability of getting an even number 3 times, then the probability of getting an odd number for odd number of times is :

(1) $\frac{1}{32}$ (2) $\frac{5}{16}$ (3) $\frac{3}{16}$ (4) $\frac{1}{2}$

Ans. (4)

Sol. ${}^nC_2\left(\frac{1}{2}\right)^n = {}^nC_3\left(\frac{1}{2}\right)^n \Rightarrow {}^nC_2 = {}^nC_3$

$\Rightarrow$ n = 5

Probability of getting an odd number for odd number of times is

$${}^5C_1\left(\frac{1}{2}\right)^5 + {}^5C_3\left(\frac{1}{2}\right)^5 + {}^5C_5\left(\frac{1}{2}\right)^5 = \frac{1}{2^5}(5+10+1)$$

$$= \frac{1}{2}$$

14. A scientific committee is to be formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then the number of ways, the committee can be formed, is :

(1) 1625 (2) 575

(3) 560 (4) 1050

Ans. (1)

Sol.

Indians	Foreigners	Number of ways
2	4	${}^6C_2 \times {}^8C_4 = 1050$
3	6	${}^6C_3 \times {}^8C_6 = 560$
4	8	${}^6C_4 \times {}^8C_8 = 15$

Total number of ways = 1625

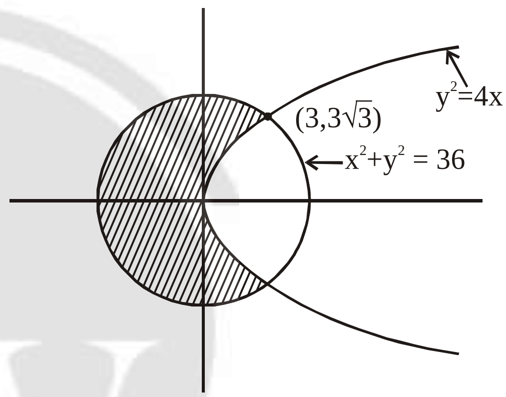
15. The area (in sq. units) of the part of the circle $x^2 + y^2 = 36$, which is outside the parabola $y^2 = 9x$, is :

(1) $24\pi + 3\sqrt{3}$ (2) $12\pi - 3\sqrt{3}$

(3) $24\pi - 3\sqrt{3}$ (4) $12\pi + 3\sqrt{3}$

Ans. (3)

Sol.



Required area

$$= \pi \times (6)^2 - 2\int_0^3 \sqrt{9}x\,dx - \int_3^6 \sqrt{36 - x^2}\,dx$$

$$= 36\pi - 12\sqrt{3} - 2\left(\frac{x}{2}\sqrt{36-x^2} + 18\sin^{-1}\frac{x}{6}\right)_3^6$$

$$= 36\pi - 12\sqrt{3} - 2\left(9\pi - 3\pi - \frac{9\sqrt{3}}{2}\right)$$

$$= 24\pi - 3\sqrt{3}$$

16. Let p and q be two positive numbers such that $p + q = 2$ and $p^4 + q^4 = 272$. Then p and q are roots of the equation :

(1) $x^2 - 2x + 2 = 0$

(2) $x^2 - 2x + 8 = 0$

(3) $x^2 - 2x + 136 = 0$

(4) $x^2 - 2x + 16 = 0$

Ans. (4)

Sol. Consider $(p^2 + q^2)^2 - 2p^2q^2 = 272$

$$((p + q)^2 - 2pq)^2 - 2p^2q^2 = 272$$

$$16 - 16pq + 2p^2q^2 = 272$$

$$(pq)^2 - 8pq - 128 = 0$$

$$(pq)^2 - 8pq - 128 = 0$$

$$pq = \frac{8 \pm 24}{2} = 16, -8$$

$\therefore$ pq = 16

$\therefore$ Required equation : $x^2 - (2)x + 16 = 0$

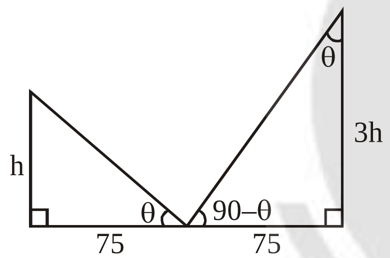
17. Two vertical poles are 150 m apart and the height of one is three times that of the other. If from the middle point of the line joining their feet, an observer finds the angles of elevation of their tops to be complementary, then the height of the shorter pole (in meters) is :

(1) $20\sqrt{3}$ (2) $25\sqrt{3}$

(3) 30 (4) 25

Ans. (2)

Sol.



$$\tan\theta = \frac{h}{75} = \frac{75}{3h}$$

$$\Rightarrow \quad h^2 = \frac{(75)^2}{3}$$

$$h = 25\sqrt{3}\text{ m}$$

18. $\lim\limits_{x \to 0} \dfrac{\int_0^{x^2} \left(\sin\sqrt{t}\right)dt}{x^3}$ is equal to :

(1) $\frac{2}{3}$ (2) $\frac{3}{2}$ (3) 0 (4) $\frac{1}{15}$

Ans. (1)

Sol. $\lim\limits_{x \to 0^+} \dfrac{\int_0^{x^2} \sin\sqrt{t}\,dt}{x^3} = \lim\limits_{x \to 0^+} \dfrac{(\sin x)2x}{3x^2}$

$$= \lim_{x \to 0^+}\left(\frac{\sin x}{x}\right) \times \frac{2}{3} = \frac{2}{3}$$

19. If $e^{(\cos^2 x + \cos^4 x + \cos^6 x + ...\infty)\log_e 2}$ satisfies the equation $t^2 - 9t + 8 = 0$, then the value of $\dfrac{2\sin x}{\sin x + \sqrt{3}\cos x}\left(0 < x < \dfrac{\pi}{2}\right)$ is

(1) $2\sqrt{3}$ (2) $\frac{3}{2}$

(3) $\sqrt{3}$ (4) $\frac{1}{2}$

Ans. (4)

Sol. $e^{(\cos^2\theta + \cos^4\theta + ...\infty)\ell n^2} = 2^{\cos^2\theta + \cos^4\theta + ...\infty}$

$= 2^{\cot^2\theta}$

Now $t^2 - 9t + 9 = 0 \Rightarrow t = 1, 8$

$$\Rightarrow \quad 2^{\cot^2\theta} = 1, 8 \Rightarrow \cot^2\theta = 0, 3$$

$$0 < \theta < \frac{\pi}{2} \Rightarrow \cot\theta = \sqrt{3}$$

$$\Rightarrow \quad \frac{2\sin\theta}{\sin\theta + \sqrt{3}\sin\theta} = \frac{2}{1 + \sqrt{3}\cot\theta} = \frac{2}{4} = \frac{1}{2}$$

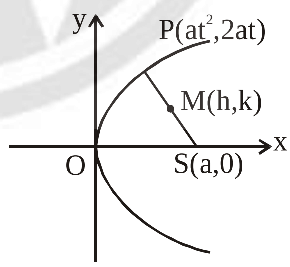
20. The locus of the mid-point of the line segment joining the focus of the parabola $y^2 = 4ax$ to a moving point of the parabola, is another parabola whose directrix is :

(1) $x = -\frac{a}{2}$ (2) $x = \frac{a}{2}$

(3) x = 0 (4) x = a

Ans. (3)

Sol.



$$h = \frac{at^2 + a}{2}, \; k = \frac{2at + 0}{2}$$

$$\Rightarrow \quad t^2 = \frac{2h - a}{a} \text{ and } t = \frac{k}{a}$$

$$\Rightarrow \quad \frac{k^2}{a^2} = \frac{2h - a}{a}$$

$\Rightarrow$ Locus of (h, k) is $y^2 = a(2x - a)$

$$\Rightarrow \quad y^2 = 2a\left(x - \frac{a}{2}\right)$$

Its directrix is $x - \frac{a}{2} = -\frac{a}{2} \quad \Rightarrow \quad x = 0$

SECTION-B

1. If the least and the largest real values of α, for which the equation $z + \alpha|z - 1| + 2i = 0$ ($z \in C$ and $i = \sqrt{-1}$) has a solution, are p and q respectively; then $4(p^2 + q^2)$ is equal to ____

Ans. (10)

Sol. Put $z = x + iy$

$x + iy + \alpha|x + iy - 1| + 2i = 0$

$$\Rightarrow \quad x + \alpha\sqrt{(x-1)^2 + y^2} + i(y+2) = 0 + 0i$$

$$\Rightarrow \quad y + 2 = 0 \text{ and } x + \alpha\sqrt{(x-1)^2 + y^2} = 0$$

$$\Rightarrow \quad y = -2 \text{ and } \alpha^2 = \frac{x^2}{x^2 - 2x + 5}$$

$$\text{Now } \frac{x^2}{x^2 - 2x + 5} \in \left[0, \frac{5}{4}\right]$$

$$\therefore \quad \alpha^2 \in \left[0, \frac{5}{4}\right] \quad \Rightarrow \alpha \in \left[-\frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2}\right]$$

$$\therefore \quad p = -\frac{\sqrt{5}}{2};\ q = \frac{\sqrt{5}}{2}$$

$$\Rightarrow \quad 4(p^2 + q^2) = 4\left(\frac{5}{4} + \frac{5}{4}\right) = 10$$

2. If $\int_{-a}^{a} (|x| + |x-2|)dx = 22, (a > 2)$ and $[x]$ denotes the greatest integer $\le x$, then $\int_{a}^{-a} (x + [x])\,dx$ is equal to _______.

Ans. (3)

Sol. $$\int_{-a}^{0} (-2x+2)dx + \int_{0}^{2} (x+2-x)dx + \int_{2}^{a} (2x-2)dx = 22$$

$$x^2 - 2x\Big|_0^{-a} + 2x\Big|_0^2 + x^2 - 2x\Big|_0^a = 22$$

$a^2 + 2a + 4 + a^2 - 2a - (4 - 4) = 22$

$2a^2 = 18 \Rightarrow a = 3$

$$\int_{3}^{-3} (x + [x])dx = -(-3 - 2 - 1 + 1 + 2) = 3$$

3. Let $A = \{n \in N : n \text{ is a 3-digit number}\}$

$B = \{9k + 2 : k \in N\}$

and $C = \{9k + l : k \in N\}$ for some l $(0 < l < 9)$

If the sum of all the elements of the set $A \cap (B \cup C)$ is 274×400, then l is equal to ______.

Ans. (5)

Sol. B and C will contain three digit numbers of the form 9k + 2 and 9k + ℓ respectively. We need to find sum of all elements in the set $B \cup C$ effectively.

Now, $S(B \cup C) = S(B) + S(C) - S(B \cap C)$

where S(k) denotes sum of elements of set k.

Also, B = {101, 109, ..., 992}

$$\therefore \quad S(B) = \frac{100}{2}(101 + 992) = 54650$$

Case-I : If $\ell = 2$

then $B \cap C = B$

$\therefore \quad S(B \cup C) = S(B)$

which is not possible as given sum is

$274 \times 400 = 109600$.

Case-II : If $\ell \neq 2$

then $B \cap C = \phi$

$\therefore \quad S(B \cup C) = S(B) + S(C) = 400 \times 274$

$$\Rightarrow \quad 54650 + \sum_{k=11}^{110} 9k + \ell = 109600$$

$$\Rightarrow \quad 9\sum_{k=11}^{110} k + \sum_{k=11}^{110} \ell = 54950$$

$$\Rightarrow \quad 9\left(\frac{100}{2}(11 + 110)\right) + \ell(100) = 54950$$

$\Rightarrow \quad 54450 + 100\ell = 54950$

$\Rightarrow \quad \ell = 5$

4. Let M be any 3×3 matrix with entries from the set {0, 1, 2}. The maximum number of such matrices, for which the sum of diagonal elements of M^TM is seven, is _______.

Ans. (540)

Sol. $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$

$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 7$

Case-I : Seven (1's) and two (0's)

$^9C_2 = 36$

Case-II : One (2) and three (1's) and five (0's)

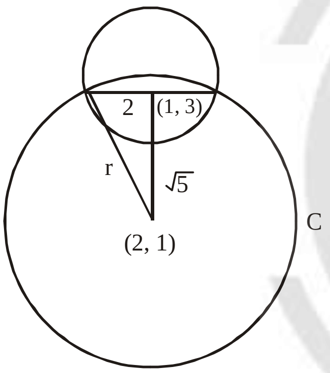
$\frac{9!}{5!3!} = 504$

$\therefore$ Total = 540

5. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord of another circle 'C', whose center is at (2, 1), then its radius is ______ .

Ans. (3)

Sol.



$x^2 + y^2 + 2x - 6y + 6 = 0$

center (1, 3)

radius = 2

distance between (1, 3) and (2, 1) is $\sqrt{5}$

$\therefore \left(\sqrt{5}\right)^2 + (2)^2 = r^2$

$\Rightarrow r = 3$

6. The minimum value of α for which the equation $\frac{4}{\sin x} + \frac{1}{1-\sin x} = \alpha$ has at least one solution in $\left(0, \frac{\pi}{2}\right)$ is _______.

Ans. (9)

Sol. Let $f(x) = \frac{4}{\sin x} + \frac{1}{1-\sin x}$

$\Rightarrow f'(x) = 0 \Rightarrow \sin x = 2/3$

$\therefore f(x)_{min} = \frac{4}{2/3} + \frac{1}{1-2/3} = 9$

$f(x)max \to \infty$

$f(x)$ is continuous function

$\therefore \alpha_{min} = 9$

7. $\lim_{n\to\infty} \tan\left\{\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{1+r+r^2}\right)\right\}$ is equal to ____.

Ans. (1)

Sol. $\lim_{n\to\alpha} \tan\left(\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{1+r(r+1)}\right)\right)$

$= \lim_{n\to\alpha} \tan\left(\sum_{r=1}^{n} \tan^{-1}\left(\frac{r+1-r}{1+r(r+1)}\right)\right)$

$= \tan\left(\lim_{n\to\alpha} \sum_{r=1}^{n}\left[\tan^{-1}(r+1) - \tan^{-1}(r)\right]\right)$

$= \tan\left(\lim_{n\to\infty}\left(\tan^{-1}(n+1) - \frac{\pi}{4}\right)\right)$

$= \tan\left(\frac{\pi}{4}\right) = 1$

8. Let three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ be such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}, \vec{a}.\vec{c} = 7$ and $\vec{b}$ is perpendicular to $\vec{c}$, where $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{k}$, then the value of $2\left|\vec{a} + \vec{b} + \vec{c}\right|^2$ is ______.

Ans. (75)

Sol. Let $\vec{c}=\lambda\left(\vec{b}\times\left(\vec{a}\times\vec{b}\right)\right)$

$$=\lambda\left(\left(\vec{b}.\vec{b}\right)\vec{a}-\left(\vec{b}.\vec{a}\right)\vec{b}\right)$$

$$=\lambda\left(5\left(-\hat{i}+\hat{j}+\hat{k}\right)+2\hat{i}+\hat{k}\right)$$

$$=\lambda\left(-3\hat{i}+5\hat{j}+6\hat{k}\right)$$

$$\vec{c}.\vec{a}=7\Rightarrow 3\lambda+5\lambda+6\lambda=7$$

$$\lambda=\frac{1}{2}$$

$$\therefore\ 2\left|\left(\frac{-3}{2}-1+2\right)\hat{i}+\left(\frac{5}{2}+1\right)\hat{j}+\left(3+1+1\right)\hat{k}\right|^2$$

$$=2\left(\frac{1}{4}+\frac{49}{4}+25\right)=25+50=75$$

9. Let B_i (i = 1, 2, 3) be three independent events in a sample space. The probability that only B_1 occur is α, only B_2 occurs is β and only B_3 occurs is γ. Let p be the probability that none of the events B_i occurs and these 4 probabilities satisfy the equations $(\alpha-2\beta)p=\alpha\beta$ and $(\beta-3\gamma)p=2\beta\gamma$ (All the probabilities are assumed to lie in the interval (0,1)). Then $\frac{P(B_1)}{P(B_3)}$ is equal to______.

An.s (6)

Sol. Let $P(B_1)=p_1$, $P(B_2)=p_2$, $P(B_3)=p_3$

given that $p_1(1-p_2)(1-p_3)=\alpha$ .....(i)

$p_2(1-p_1)(1-p_3)=\beta$ .....(ii)

$p_3(1-p_1)(1-p_2)=\gamma$ .....(iii)

and $(1-p_1)(1-p_2)(1-p_3)=p$ .....(iv)

$$\Rightarrow\ \frac{p_1}{1-p_1}=\frac{\alpha}{p},\ \frac{p_2}{1-p_2}=\frac{\beta}{p}\ \ \&\ \ \frac{p_3}{1-p_3}=\frac{\gamma}{p}$$

Also $\beta=\frac{\alpha p}{\alpha+2p}=\frac{3\gamma p}{p-2\gamma}$

$\Rightarrow$ $\alpha p-2\alpha\gamma=3\alpha\gamma+6p\gamma$

$\Rightarrow$ $\alpha p-6p\gamma=5\alpha\gamma$

$$\Rightarrow\ \frac{p_1}{1-p_1}-\frac{6p_3}{1-p_3}=\frac{5p_1p_3}{(1-p_1)(1-p_3)}$$

$\Rightarrow$ $p_1-6p_3=0$

$$\Rightarrow\ \frac{p_1}{p_3}=6$$

10. Let $P=\begin{bmatrix}3&-1&-2\\2&0&\alpha\\3&-5&0\end{bmatrix}$, where $\alpha\in R$. Suppose $Q=[q_{ij}]$ is a matrix satisfying $PQ=kI_3$ for some non-zero $k\in R$. If $q_{23}=-\frac{k}{8}$ and $|Q|=\frac{k^2}{2}$, then α^2+k^2 is equal to _______.

Ans. (17)

Sol. $PQ=kI$

$|P|.|Q|=k^3$

$\Rightarrow$ $|P|=2k\neq 0\Rightarrow$ P is an invertible matrix

$\because$ $PQ=kI$

$\therefore$ $Q=kP^{-1}I$

$$\therefore\ Q=\frac{adj.P}{2}$$

$$\because\ q_{23}=-\frac{k}{8}$$

$$\therefore\ \frac{-(3\alpha+4)}{2}=-\frac{k}{8}\Rightarrow k=4$$

$\therefore$ $|P|=2k\Rightarrow k=10+6\alpha$ ...(i)

Put value of k in (i).. we get $\alpha=-1$